Implementation of Longitudinal Control Algorithm for Vehicle Merging

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Abstract
This paper presents a real-time implementation of a rather general adaptive merging algorithm for automated highway systems. A merging control problem is proposed first. A real-time adaptive closed-loop algorithm is then presented, which is used to calculate a smooth reference speed trajectory for the merging vehicle based on the speed of the main lane vehicle. This algorithm can also be applied even when the main lane vehicles change speed. To make the algorithm adapt to different road layouts and to increase safety, a concept of virtual platooning is proposed. It effectively shifts the time of platoon formation forward prior to the start of real merging. Aspects closely related to real-time implementation are discussed. Test results are presented and briefly analyzed.

1 Introduction
Automated highway systems (AHS) offer the potential of significant improvements in traffic flow volume and stability, as well as safety, when compared with current manually-controlled highway driving (Shladover, 1991). Implementation of AHS requires completely automated lateral and longitudinal control of vehicle motions as well as coordination of the maneuvers of different vehicles. Considerable success has already been demonstrated in achieving high-performance automatic lateral (Peng and Tomizuka, 1994; Tan et al., 1999) and longitudinal (Hedrick, 1998; Lu et al., 2000) control of full-scale road vehicles on test track.

The merge maneuver requires a combination of decisions and control actions at both the regulation layer and the coordination layer of the five-layer control hierarchy that Varaiya defined for AHS (Varaiya, 1993). Current thinking about AHS operations favors the addition of vehicles entering at a merge junction on to the back end of a passing platoon of vehicles for reasons of safety and operating simplicity. However, a more general operating condition, which could provide higher efficiency in high-density traffic conditions, would permit entering vehicles to be inserted into the middle of a passing platoon. This more challenging case has been implemented here to show that it is feasible. The simpler case (attaching to the back of a passing platoon) requires only a subset of the capability demonstrated here.

Most generally, vehicle merging can be abstracted as the problem of one vehicle from the entry lane merging between two vehicles traveling in a platoon in the main lane. Other situations are all special cases of this general case. Generally, a roadside control computer at the merge junction would perform the coordination layer tasks, but in the experimental implementation described here those functions were performed by the lead vehicle in the main lane. These tasks involve:

(a) the determination of vehicle ID of the vehicles in the platoon before and after the merging maneuver
(b) the selection of two vehicles in the main lane between which the merging vehicle is to enter (based on the traffic situation in both main lane and merging lane and the road geometry)
(c) the coordination between the platoons in the main lane
(d) passing relevant information to each vehicle.

The regulation layer tasks are fulfilled by each vehicle itself, and always involve the lower-level control of throttle, brake and steering actuation. In some cases, they may also involve generating higher-level control commands based on reference trajectories (although these may be considered as coordination-layer activities in some implementations).

Prior research on vehicle merging has addressed the design of the coordination layer protocols (Antoniotti et al., 1999) for simulation. Related research works are referred to (Narendran et al., 1992). However, the problem addressed and the solution obtained here are different.

This paper concentrates on the implementation of a newly developed algorithm for general merging of vehicles into a highway (Lu and Hedrick, 2000a). This completely new real-time closed-loop adaptive merging algorithm generates a smooth reference trajectory for the merging vehicle according to the speed of the leader vehicle in the platoon in the main lane. It is not only suitable for two typical road layouts which represent most practical cases but is also applicable to the difficult situation when the speed of the platoon $v_p(t)$ in the main lane is changing with respect to time.
Part of the main idea here is to introduce the concept of virtual platooning which effectively shifts the time of platoon formation forward prior to the start of real merging. This is the highlight of the algorithm, and is extremely important for real-time implementation and safety.

One vehicle merging in between two other vehicles requires that at the time instant of merging $T_{merg}$, the following two conditions are to be satisfied:

(i) \[ v(T_{merg}) = v_p(T_{merg}) \]
\[ a(T_{merg}) = a_p(T_{merg}) \]

(ii) The relative distance between the merging vehicle and either of the two vehicles of the platoon in the main lane is approximately the desired following distance $l_{des\_follow}$.

This algorithm has been successfully implemented and tested on the Richmond Field Station (RFS) low speed track with magnets installed every one meter apart at PATH, U. C. Berkeley. This track, which was designed for different test purposes, poses several geometric challenges, making the merge implementation considerably more difficult than it would be in a normal highway environment.

2 Adaptive Merging Algorithm

This section proposes the longitudinal control problem for vehicle merging and an adaptive algorithm. The real-time merging maneuver has two main requirements:

(1) a splitting maneuver is performed by the two vehicles in the main lane after merging starts ($t \geq T_{merg}$) so that the distance between these two vehicles $l_0(=2l_{des\_follow} + l_2)$ is achieved and maintained;

(2) the speed of the merging vehicle is properly adjusted in real time according to the speed of the leader vehicle in the main lane and the relative distance of each vehicle from the merging point.

Pertinent factors for the merging maneuver can be divided into three parts: (a) geometric layout of the road, (b) absolute speed and relative positions of relevant vehicles in both main lane and merging lane, and (c) signal measurement on each vehicle and communication between relevant vehicles.

2.1 Nomenclature

Variables and parameters used in the longitudinal control algorithm for merging are listed below:

$P_i, i = 1, 3$—vehicle ID on main lane

$P_2$—vehicle ID on merging lane

$v(t)$—merging vehicle speed, measurable

$v_{md}(t)$—desired speed of merging vehicle, to be determined

$v_p(t)$—speed of the leader vehicle in the platoon on main lane, measurable

$x(t), x_{md}(t)$—relative distance and reference relative distance between two vehicles, which means the virtual distance between the leader vehicle and the merging vehicle here

$t_{10}$—time instant for $P_1$ passing the virtual starting point on main lane

$t_{20}$—time instant for $P_2$ passing the virtual starting point on merging lane

$t_{merg} = \max(t_{10}, t_{20})$—time instant for longitudinal merging control algorithm to start

$T_{merg}$—time instant when merging is completed

$T_{virt}$—time instant when a virtual platoon is formed but merging is not completed

$Q_{start\_1}, Q_{start\_2}$—vehicle positions on main lane and merging lane when $t = t_{merg}$, respectively

$Q_{virt\_1}, Q_{virt\_2}$—virtual positions in main lane and merging lane at which virtual platoon is formed before practical vehicle merging happens

$l_0$—desired distance between the two consecutive vehicles in the main lane (for the merging vehicle to enter)

$l_{des\_follow}$—desired distance between consecutive vehicles in platoon after merging

$l_i$—length of vehicle $P_i$, $i = 1, 2, 3$.

$Q_1$—a point in main lane marked by infrastructure of magnets using special coding

$Q_2$—a point in merging lane marked by infrastructure of magnets using special coding

$Q_0$—real crossing (merging) point of main lane and merging lane

$|Q_1Q_0|$—the distance between $Q_1$ and $Q_0$.

![Figure 1: Road layout A](image-url)

Figure 1: Road layout A
2.2 Geometric Layout of the Road

Two different geometric road layouts, which represent most of the possible road layouts in practice, lead to slightly different problem formulation.

In the first layout, there is no parallel lane for the merging vehicle to accelerate, which is called layout $A$ as in Fig. 1. The test track layout in Richmond Field Station at U. C. Berkeley and that at Crows Landing are of this type. In fact, as long as the merging point is fixed, which is the intersection of the merging lane and the main lane, the road layout always belongs to this type.

$Q_1, Q_2, Q_0$ are marked by infrastructure such as specially coded magnets or transponders. Once vehicle $P_1$ or $P_2$ passes them, it will communicate this to the corresponding vehicle, which indirectly tells time and position. Once $P_1$ and $P_2$ have passed $Q_1$ and $Q_2$ respectively, merging starts. For such scenario, since the merging point $Q_0$ is fixed, the merging vehicle needs to arrive this point at the right time instant and at proper speed and acceleration.

In a freeway, there is usually a short lane parallel to the main lane for the merging vehicle to accelerate, which is called layout $B$ as in Fig. 2.

Here, $Q_1$ and $Q_{01}$ on the main lane as well as $Q_2$, $Q_{02}$ on the merging lane are again marked by infrastructure such as specially coded magnets. This road layout has more flexibility because merging can be carried out at any point between $Q_{01}$ and $Q_{02}$ in a longer time period compared to the previous road layout. It is thus more flexible to adjust the merging time instant as well as the speed and acceleration of the merging vehicle in this layout.

It is clear that a control algorithm suitable for road layout $A$ will be automatically applicable to the less challenging road layout $B$.

2.3 Mathematical Modelling for Merging

The main longitudinal control problem for vehicle merging from a control viewpoint is to determine a desired reference speed $v_{md}(t)$ of the merging vehicle. Any robust controller with reasonably good performance can be used to track this reference speed. The merging problem becomes a trajectory planning problem for the merging vehicle. $v_{md}(t)$ is determined by the speed of the platoon $v_p(t)$ in the main lane as well as the positions of the relevant vehicles when merging starts.

2.3.1 Longitudinal Control Problem for Merging

Design a reference trajectory $v_{md}(t)$ for merging vehicle $P_2$ such that

$$\begin{align*}
  v_{md}(t_{merg}) &= v(t_{merg}) \\
  v_{md}(T_{merg}) &= v_p(T_{merg}) \\
  a_{md}(T_{merg}) &= a_p(T_{merg})
\end{align*}$$

(2) three vehicles $P_1, P_2, P_3$ form a platoon of speed $v_p(t)$ for $t \geq T_{merg}$

(3) the relative distance between two consecutive vehicles is the same as the prescribed distance $l_{des\_follow}$ for $t \geq T_{merg}$.

The following conditions are assumed:

(a) The platoon of vehicles $P_1$ and $P_3$ will split to a prescribed distance $l_0 = 2l_{des\_follow} + l_2$ and maintain this distance by a separate controller around the time instant of real merging;

(b) $v_p$ is measured by $P_1$ and is passed to $P_2$ and $P_3$ by communication;

(c) The distances $|Q_1Q_0|, |Q_2Q_0|, |Q_1Q_{02}|$ and $|Q_2Q_{02}|$ are known in advance and

$$|Q_{start\_1}Q_0| - |Q_{start\_2}Q_0| + l_1 + l_{des\_follow} > 0.$$  

(d) The time instant $t_{01}$ when $P_1$ passes $Q_1$ is measured by $P_1$ and is passed to $P_2$ by communication. The time instant $t_{02}$ when $P_2$ passes $Q_2$ is measured by $P_2$ itself. Thus the merging starting time instant $t_{merg}$ and position $Q_{start\_1}$ and $Q_{start\_2}$ can be calculated.

2.3.2 Unified Modeling

It is clear that, if a merging algorithm works for road layout $A$, it is also applicable to road layout $B$. For road layout $A$, the concept of the virtual platoon is proposed. The main idea is to form a virtual platoon before $P_1$ and $P_2$ arrive at the merging point $Q_0$. A virtual platoon means that at some time instant $T_{virt}$

$$t_{merg} \leq T_{virt} \leq T_{merg}$$

$P_1$ arrives at $Q_{virt\_1}$ as shown in Fig. 3.

At this point, the following conditions are satisfied.

$$\begin{align*}
  v_{md}(T_{virt}) &= v_p(T_{virt}) \\
  a_{md}(T_{virt}) &= a_p(T_{virt}) \\
  |Q_{virt\_1}Q_0| + l_1 + l_{des\_follow} &= |Q_{virt\_2}Q_0|
\end{align*}$$
2.4 Closed-loop Adaptive Solution

The following is a real-time closed-loop adaptive solution of \( v_{md} \) in (2.1). The solution is of a variable structure and has the same smoothness property as \( v(t) \). This point is important in implementation when calculating \( \frac{d}{dt}(v_{md}(t)) \) in real-time.

**Theorem** Suppose that

1. Time response of the known longitudinal controller is fast enough and speed and distance tracking error is small enough, i.e.
   \[
   v(t) \approx v_{md}(t), \quad x(t) \approx x_{md}(t)
   \]
   for \( t \in [t_{merg}, T_{merg}] \);

2. \( |Q_1 Q_0| \) and \( |Q_2 Q_0| \) are large enough, or equivalently, the time interval \([t_{merg}, T_{merg}]\) is sufficiently large to adjust the speed and distance of the merging vehicle;

3. The starting point for merging and vehicle length have the following relation
   \[
   \text{dist}_\text{para} = |Q_{\text{start}-1} Q_0| - |Q_{\text{start}-2} Q_0| + l_1 + l_{\text{des, follow}} > 0
   \]

4. There exists \( \delta > 0 \) such that the following condition is true for most of the time
   \[
   v_p(t) \geq v(t_{merg}) + \delta, \quad t \in [t_{merg} + \eta, T_{virt}]
   \]
   where \( 0 < \eta \ll 1 \); This means that there exists a constant \( \epsilon > 0 \) such that
   \[
   \int_{t}^{t+\epsilon} (v_p(t) - v(t_{merg})) dt \geq \epsilon \delta
   \]
   as long as \([t, t + \epsilon] \subset [t_{merg} + \eta, T_{virt}]\).

5. The following reference speed is used for the longitudinal control of \( P_2 \)
   \[
   v_{md}(t) = \left\{
   \begin{array}{ll}
   (1 - \alpha(t)) & v(t_{merg}) + \alpha(t) v_p(t), \quad t_{merg} \leq t \leq T_{virt} \\
   v_p(t), & T_{virt} < t < T_{merg}
   \end{array}
   \right.
   \]
   \[
   \alpha_0(t) = \frac{\int_{t_{merg}}^{t} v_p(s)ds + |Q_{\text{start}-1} Q_0| - |Q_{\text{start}-2} Q_0| + l_1 + l_{\text{des, follow}}}{\int_{t_{merg}}^{t} \alpha_0(t) dt}, \quad \beta > 0
   \]

Then virtual platooning is guaranteed to be formed for some \( \beta > 0 \). i.e. there exist a constant \( \beta > 0 \), a time instant \( T_{virt} \subset [t_{merg}, T_{merg}] \) and points \( Q_{\text{virt},1} \) and \( Q_{\text{virt},2} \) such that

\[
|Q_{\text{virt},i} Q_0| \leq |Q_i Q_0|, \quad i = 1, 2
\]

and all the conditions in (2.1) are satisfied.

**Remark 2.1** The physical meaning of the algorithm and the conditions in the theorem are briefly explained as follows.

(a) The reference trajectory can be divided into two phases according to time:

- **Phase 1:** \( t_{merg} \leq t \leq T_{virt} \)
  - \( v(t) \) is constant and equal to \( v(t_{merg}) \).
  - \( x(t) \) is linearly increasing with a constant rate.

- **Phase 2:** \( T_{virt} < t < T_{merg} \)
  - \( v(t) \) is constant and equal to \( v_p(t) \).
  - \( x(t) \) is linearly increasing with a constant rate.

The physical meaning is briefly explained as follows. The first equation is required by the smoothness of the trajectory. The second, the third and the fourth equations are related to the compatibility conditions of speed and distance. The last two conditions are virtual platooning requirements. There is no doubt that a human-driven vehicle does not need to satisfy this complete set of conditions in order to merge successfully. It is suggested that, for merging maneuvers, the \( v_{md} \) of an automated vehicle should satisfy these conditions for safety and smoothness.
Phase 1. \( t_{\text{merg}} \leq t < t_{\text{virt}} \), this is the essential part of the merging trajectory planning. The purpose is to adjust both speed \( v(t) \) and distance \( |Q_{\text{virt},1}Q_0| \) such that the formation of the virtual platoon is completed at the end of this phase.

Phase 2. \( t_{\text{virt}} \leq t \leq t_{\text{merg}} \), virtual platooning control. As long as the merging vehicle follows the speed and acceleration of the platoon in the main lane, real merging will be guaranteed at time instant \( t_{\text{merg}} \).

(b) The assumptions in the theorem are basically the physical constraints which can be set as required in practice. Condition (1) depends on the controller adopted. Condition (2) and (3) can be implemented by proper layout of the infrastructure (such as magnet coding, roadside transponder, or GPS and map signals for vehicle locations) in both merging lane and main lane. Condition (4) indicates that the overall platoon speed should be higher than the speed of the merging vehicle during phase 1 of merging, which is usually true or can be made true by suitably controlling the speed of the merging vehicle.

**Remark 2.2** The parameter \( \beta \) determines the length of time period in phase 1. In general, larger \( \beta \) will lead to shorter phase 1, i.e., the virtual platoon is formed earlier. However, this will lead to higher acceleration demands. Usually, choosing \( \beta \in [1.5, 5.0] \) will be sufficient. The larger the speed difference

\[
v_p(t) - v(t_{\text{merg}}), t \in [t_{\text{merg}} + \eta, t_{\text{merg}}]
\]

is, the sooner the virtual platoon will be formed.

### 3 Real-time Implementation

This section addresses some practical issues in the real-time implementation of longitudinal control for vehicle merging. In particular, the magnet-based infrastructure reference system (Tan et al., 1999) is used for the example in this paper.

#### 3.1 Magnet Observer Based Longitudinal Position Information

For real-time implementation of the merging maneuver, a key point is the knowledge of the relative distance between vehicles and the distance to the merging point. In this paper, magnetometer and speedometer measurements are used to estimate the absolute moving distance of the merging vehicle and the platoon vehicles. The merging vehicle needs to acquire at any time instant \( t \in [t_{\text{merg}}, t_{\text{merg}}] \) its absolute speed and relative distance with respect to the merging point. The two relevant vehicles in the main lane need to obtain their absolute speed and distance with respect to the merging point as well as their relative distance at any time instant \( t \in [t_{\text{merg}}, t_{\text{merg}}] \). Radar measurement is obviously not suitable for the merging vehicle.

The magnets in both main lane and merging lane are installed at a regular distance \( \text{mag.dist} \) with maximum error bound \( \text{err}_{\text{mag}} \). A simple approach to get a continuous position measurement from the magnets is to fuse the discrete magnet measurement with the continuous speed measurement. Considering the possibility that the magnetometer could miss some magnets, the fusion of magnet distance is only carried out when such a miss does not occur. After each magnet updating, the time parameter is reset to 0. From this time instant, the temporary moving distance of the vehicle is

\[
\text{temp.move.dist} = \int_0^{t_1} v(s) \, ds
\]

where the time instant \( t_1 \) is when a new magnet is measured. Clearly, if no magnet is missed during this period, then there should be

\[
\text{temp.move.dist} \approx \text{mag.dist} + \text{err}_{\text{mag}}.
\]

Otherwise, it should be

\[
\text{temp.move.dist} \approx m (\text{mag.dist} + \text{err}_{\text{mag}})
\]

for some integer \( m \) which is the number of magnets missed. Now the absolute distance is

\[
\sum \text{temp.move.dist}.
\]

This estimation can also be used to calculate the relative distance between \( P_1 \) and \( P_2 \) in the main lane when radar reading is not appropriate and the virtual relative distance between \( P_1 \) and \( P_2 \).

Real-time application of this algorithm was successfully tested at low speed at low and high speeds. Even if some magnets are missed, the distance measurement is still reasonably good.

#### 3.2 Signal Measurement and Communication

Vehicle speed measurement relies on fusion of data from 4 wheel speed sensors on each vehicle. Real-time communication between vehicles is crucial for speed profile calculation and synchronization. This is different from the basic platoon operation where all the distances between consecutive vehicles can also be measured by radar. For merging, the virtual relative positions are calculated from the data passed by communication. WaveLan radio is used which is strong enough for communication between vehicles over 200m apart and thus suitable for merging communication at the PATH RFS test site. The information passed is: \( v \)– speed, magnet counter, special magnet coding and maneuver ID.

Radar is not used in the merging vehicle (\( P_2 \)) during the merging maneuver. It is used in the following car


3.3 Safety Consideration

Safe merging depends on many reliability factors, including both software and hardware. Hardware reliability is related to sensors, actuators, physical condition of the vehicle, communication facilities, road environment conditions, etc., which are not discussed here. Some important safety factors closely related to control design and implementation are:

1. Accuracy of controller;
2. Reliability of the splitting maneuver for the second vehicle in the main lane;
3. Reliability of the merging algorithm for the merging vehicle: This is crucial because each controller as well as vehicle has certain physical restrictions, such as maximum acceleration and deceleration. In practice, due to the road infrastructure the desired acceleration

\[ a_{md}(t) = v_{md}(t) \]

could be too large for the merging vehicle to reach. Therefore so that the stability could be destroyed. To avoid this, some safety measure should be taken:

3-a) Forming a virtual platoon before the merging vehicle actually arrives at the merging point is the most important measure. To ensure this physically, one can install a specially magnet code for a sufficient distance before the merging point in each lane to tell the relevant vehicles in advance if the virtual platoon is actually formed as expected. If not, emergency steps should be taken to avoid crashes. Suppose the distance between the magnet and the merging point on the main lane is \( l_{safe} \). The distance between the special magnet to the merging point on the merging lane should be set as \( l_{safe} + l_{des \_ follow} + l_1 \). For safe merging, around the time instant when the leader vehicle passes the specialy coded magnet on the main lane, the merging vehicle should pass the corresponding magnet on the merging lane. The leader car can pass this message to the merging vehicle, which will decide if the merging is successful by comparing the two time instants.

3-b) Physical arrangements of the special coded magnets in both lanes which tell relevant vehicles when merging starts should be arranged to make \( \frac{dv}{dt} (v_{md}(t)) \) as small as possible.

3-c) In principle, larger \( \beta \) will lead to larger \( \frac{dv}{dt} (v_{md}(t)) \). It should also be chosen such that \( \frac{dv}{dt} (v_{md}(t)) \) is sufficiently small.

3-d) The maximum speed for relevant vehicles in the main lane is restricted within certain range.

4) Fault management: Merging faults may come from two aspects: (a) The main lane vehicles fail to split properly; (b) The merging vehicle fails to satisfy any of the requirements in distance, speed and acceleration. The fault detect and management techniques for platooning maneuvers, such as splitting, developed previously and under development for vehicle platooning can still be used for the main lane vehicles. Thorough consideration of fault management for the merging maneuver will be addressed in future work. Primary suggestion would include the following strategies.

4-a) If communication range permits, one can install \( Q_1 \) and \( Q_2 \) further away from the merging point. Now the controller can be designed such that the virtual platoon is to be formed before \( P_1 \) reaches the middle point \( \frac{1}{3} \mid Q_1 Q_0 \) and \( P_2 \) reaches the middle point \( \frac{1}{2} \mid Q_2 Q_0 \). i.e.

\[ \mid Q_{virt \_ 1} Q_0 \mid \leq \frac{1}{2} \mid Q_1 Q_0 \mid \]
\[ \mid Q_{virt \_ 2} Q_0 \mid \leq \frac{1}{2} \mid Q_2 Q_0 \mid \]

If the merging maneuver fails in the first half of the roads, one can repeat the merging maneuver in the second half. This will increase the chance of success.

4-b) If the geographical situation of the road infrastructure does not permit (4-a), the merging vehicle can be stopped manually or automatically in emergency before \( P_2 \) reaches \( Q_0 \) to abort the merging maneuver.

4-c) Similarly, one can design a splitting controller such that there is a distance of \( l_0 = 2l_{des \_ follow} + l_2 \) between \( P_1 \) and \( P_3 \) before \( P_1 \) reaches the middle point \( \frac{1}{3} \mid Q_2 Q_0 \). If this fails, one can repeat the splitting maneuver before \( P_1 \) reaches the merging point. If the splitting maneuver still fails, the merging maneuver needs to be aborted by emergency stopping the merging vehicle.

3.4 Longitudinal Controller

The longitudinal controller adopted in vehicle merging is the optimal dynamic back-stepping sliding surface control (Lu et al., 2000), which is particularly good for distance control. This method combines the following features in nonlinear control design:

1. Sliding mode method: in ideal sliding mode, dynamics of the closed-loop system are restricted to a sliding manifold (DeCarlo et al., 1988; Utkin, 1992). This approach is generally recognized to be robust to external uncertainties on this sub-manifold (Lu and Spurgeon, 1997).

2. Back-stepping is used in control design logic. In each step, a proper sliding surface is chosen. Thus multiple sliding surfaces naturally result in the end. This is a promising way of dealing with additive unmatched uncertainties in nonlinear models (Swaroop et al., 1997).

3. The sliding gain choice is based on nonlinear \( H_\infty \) approach, which can be characterized as disturbance attenuation with internal stability and with certain
performance index optimized (Lu and Hedrick, 2000b; Lu et al., 2000).

(4) Integral filters are used to calculate the derivatives of the reference state (signal) at each time step. Thus analytic differentiation is avoided, which also effectively avoids the explosion of the number of terms (Lu and Hedrick, 2000c).

4 Test Results

This section presents the results of tests conducted on the RFS test track at U. C. Berkeley. The track layout has the general topology of Fig. 1, but the merge lane includes a 90 degree curve and the main lane includes multiple reverse curves. The starting positions for the leader car on the main lane can be chosen as 170.0m ~ 190.0m away from merging point, and the starting position for the merging car in the merging lane is fixed as 153m away from the merging point. The second car in the main lane starts from any point between 8.0m to 15.0m behind the leader car as long as its radar can catch the leader car for distance initialization (on a section of track with multiple curves). All cars start to move at the same time, synchronized by communication. This is an artifact of the limited space available at the test track, which necessitates minimizing the uncertainties in the initial conditions. The merging algorithm could be implemented in a real roadway environment with sufficient maneuvering space without this start-up synchronization.

The desired gap between the two cars in main lane when the merging vehicle enters is 20.0m. The maximum speed for the leader vehicle was set to 21.0 km/h (Fig. 4) and 27.6 km/h (Fig. 5) respectively.

It can be observed from the figures that the tests have the same qualitative behavior. Fig 4. is used as an example for brief explanation.

(1) Car reference speed for the merging car is quite different from the leader car reference speed profile. For \( t \in [0, 37] \), all the cars are stationary. For \( t \in [37, 40] \), merging car follows the leader car. During this period of time, both distance control and speed are involved. This ensures that when the merging algorithm is activated at \( t = 40 \) the speed profile for the merging car is continuous. For \( t \in [40, 55] \), the merging vehicle has much lower acceleration compared to the leader car. For \( t \in [55, 75] \), the merging car gradually increases its speed to catch up to the speed of the leader car. Around \( t = 75 \) sec., virtual merging is accomplished. During the time period \( t \in [40, 75] \), only speed control is involved in the algorithm, while distance is used as logical guard. The distance error in this period is simply the integration of the speed error. Afterwards, the merging car simply follows the leader car. In the last period, both distance control and speed control are involved.

(2) There are in fact three transient phases for speed and distance errors. This is due to the change of control strategies as stated in (1). So the steady state behavior is not prominent.

Comparing Fig. 5 to Fig. 4, there are some similarities. However, a difference is that, due to the limited length of the test track, run time decreases with the increase of the maximum speed of the leader car \( P_1 \). This implies that with higher maximum speed of \( P_1 \), merging maneuver need to be accomplished in a shorter period of time. This is listed in the following table.

<table>
<thead>
<tr>
<th>max speed</th>
<th>( T_{merg} )</th>
<th>( T_{virt} )</th>
<th>( T_{virt} - T_{merg} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>21.0km/h</td>
<td>40</td>
<td>78</td>
<td>38 sec</td>
</tr>
<tr>
<td>27.6km/h</td>
<td>32</td>
<td>60</td>
<td>28 sec</td>
</tr>
</tbody>
</table>

This also shows the flexibility and capability of the merging algorithm.

![Figure 4: RFS test track, Maximum speed 21.0km/h](image)

5 Concluding Remarks

This paper describes the implementation of a general adaptive closed-loop algorithm for vehicle merging. For safety and for the unification of merging algorithms for different road layouts, the concept of virtual merging is proposed. A virtual platoon in the following sense is formed before the merging vehicle arrives at the real merging point:

(1) Speed and acceleration of the merging vehicle in the merging lane are the same as those of the platoon vehicles in the main lane;

(2) The distance from the merging point to the merging vehicle in the merging lane is the same as distance from the "merging slot" in the platoon in the main lane to the merging point.


Figure 5: RFS test track, Maximum speed 27.6$m$/h

There are two main tasks in the implementation of safe vehicle merging: (a) to split the two relevant vehicles in the main lane to a prescribed safe distance for the merging vehicle to enter; (b) to adaptively generate a smooth reference speed $v_{md}(t)$ for the merging vehicle, based on the speed of the leader vehicle in the platoon, the time instant and the relative positions with respect to the merging point of each vehicle when merging starts, and the positions of specially coded magnets in both main lane and merging lane. This algorithm has been successfully tested using automated cars with magnet-based speed and steering control. Further work will be the implementation of transition control and fault management.

6 Acknowledgment

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