Longitudinal Maneuver Design in Coordination Layer for Automated Highway System

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Abstract

The Automated Highway System (AHS) architecture of the California PATH program organizes traffic into platoons of closely spaced vehicles and has a layered control system. Maneuvers in AHS include join, split, lane change and merging/exit to and from the automated lanes. The main function of the coordination layer is to coordinate all the maneuvers of each vehicle in a specified section of AHS such that they are performed in safe, smooth and deterministic manner. Coordination layer designs maneuver by determining maneuver parameters and calculate reference trajectory for each vehicle, which keeps the feasibility of maneuvers and guarantees the safety of passengers, ride quality avoiding control saturation. Furthermore it is desired that the decisions are made in an optimal manner to minimize time and/or other performance criteria like fuel consumption while constraints on variables are not violated. This paper presents a mathematical model and design procedure for coordination layer control using a hybrid system approach which unifies coordination layer and regulation layer control. It is shown that this problem can be formulated as a mathematical programming problem, which can be easily modeled using AMPL modeling language software and readily solved by MINOS solver.

1. Introduction

The idea of aggregation of vehicles into platoons (a single vehicle is a platoon of length 1), which are controlled by a layered control system, in California PATH is to solve traffic congestion problem and increase the efficiency of existing infrastructure [7]. Maneuvers of platoons in AHS address issues such as automatic join (platoon formation), split, lane change of individual vehicles and merging(entry)/exit of vehicles to and from the automated lanes, etc. These maneuvers must go hand-in-hand with system level decision making (coordination layer) to allow them to be performed in safe, smooth and deterministic manner [11].

One of the most important tasks for the control of vehicles in automated maneuvers is trajectory planning which is feed-forward control. This can be easily seen if we notice that the reference trajectory including a step change in desired velocity can lead to high control action and result in actuator saturation or poor ride quality. Saturation means the loss of controllability and may cause the closed-loop control to be unstable. This is particularly bad to heavy duty vehicles because their acceleration capability is very low at high speed. In addition, this may lead to the failure of the maneuver and endanger the safety of the system.

To alleviate this, a proper velocity trajectory should be designed which allows for the smooth acceleration and/or deceleration of the vehicle. A smooth and continuous (or continuously differentiable) velocity trajectory guarantees such properties and reference position trajectory can also be obtained easily by integration of the velocity profile. The trajectory must be designed to keep the acceleration to be continuous and jerks in the acceleration within acceptable limit. Furthermore it is desired that the trajectory is determined in an optimal manner to minimize time and/or other performance while constraints on state variables are not violated. It should be noticed that, since all the maneuvers are performed in real-time in AHS, the algorithm for trajectory planning must be able to produce a prompt result. In this paper, we focus on the design of longitudinal maneuvers.

2. Coordination Layer : Maneuver Design

A coordination layer for AHS is to coordinate all the maneuvers of all the vehicles in a specified section of the AHS within the communication range. A maneuver can be requested randomly by a vehicle according to the needs of its passenger(s) but needs to be coordinated by the coordination layer to improve the traffic conditions [11]. To coordinate maneuvers, it is necessary to design a continuous (or continuously differentiable) speed trajectory for the vehicle in deterministic manner.

Since, from the Newton’s 2nd law, the vehicle acceleration is linearly related to the applied torque from the engine and brake, it is expected that the magnitude of control action will depend on the magnitude of the desired reference acceleration. However, assuming the stable controller is designed in regulation layer control design stage to produce control action for tracking the desired trajectory within the allowable error bounds, it was shown that the robust stability problem of controller and the trajectory design problem can be treated separately [4, 12]

The objective of general trajectory planning in AHS is to design a time varying velocity profile for the vehicle performing a designated maneuver. The desired trajectory for the joining vehicle, for example, must depend on the motion of the target platoon and the road grade which limits the acceleration capability of vehicle. If the velocity of a target platoon is known, the design of reference trajectory is relatively easy, however, in general, the problem is very difficult since it is known only in real-time, not in advance. This is the case for heavy-duty trucks in practical road situation. If the platoon acceleration is constant or zero, a smooth trajectory which is not violating the given acceleration and jerk limits can be easily designed. Taking into account the physical limit of vehicle (actuator limit) and road
condition such as road grade adds another dimension of complexity in maneuver design.

Safe-guaranteed closed-loop type velocity profile has been developed in PATH [4,6]. However, to get a feasible solution for motion planning of moving object, it is a common approach to design an open-loop trajectory and use feed-back controller to track the reference trajectory. Several useful open-loop trajectory design methods have already been proposed [2,9,10].

Existing trajectory design methods produce reference trajectories which guarantee the safety of platoon maneuver or which are optimal in some sense. However they focused on the design of trajectory for single vehicle only and rarely consider the environment for a certain maneuver, which effects on the feasibility of the maneuver design.

3. Mathematical Models

AHS can be described as a type of hybrid control system and a unified model for coordination and regulation layer was proposed in [11].

3.1 Regulation Layer

The regulation layer in AHS is composed of vehicle dynamics and controller. The regulation layer controller takes care of the vehicle dynamics including engine, power-train, drive-train, and tire dynamics. A simplified longitudinal dynamics model for a ground vehicle is given:

\[ \ddot{x} = v \]  
\[ \dot{v} = \frac{1}{M}(F_t - F_b - F_a - F_r - F_g) \]  

where \( x \) and \( v \) are position and speed of the vehicle respectively, and the terms in (2) are defined as:

\( F_t \) : tractive force,

\( F_b \) : brake force ,

\( F_a = \lambda \cdot A \cdot v^2 \) : aerodynamic drag force,

\( F_r = \mu \cdot M \cdot g \cdot \cos \theta \) : rolling resistance,

\( F_g = M \cdot g \cdot \sin \theta \) : grade resistance,

\( \lambda \) : drag coefficient,

\( A \) : projected frontal area of vehicle,

\( M \) : vehicle mass,

\( \theta \) : road grade.

Figure 1 shows the typical performance curve of an automotive engine. Since the engine power is passed to driving wheel through transmission and drive-line, a particular engine speed corresponds to a particular vehicle speed, and since the tractive force is proportional to the engine torque, if tire slip can be ignored, the variation of the tractive force with the variation of the vehicle speed will depend upon the variation of the engine torque with the variation of engine speed. Hence the curve showing the relation between tractive force and the vehicle speed will be of the same shape; in fact, the same curve might be used provided that the scales were suitably selected [3].

From the above reasoning, if we assume that the torque curve can be approximated by a function of engine speed, considering the linear relation between the vehicle velocity and engine rpm (for a specific gear ratio), we can conclude that the torque is also a function of vehicle speed then, from (2), the acceleration capability of a vehicle becomes a function of vehicle speed, road grade, and vehicle mass.

Generally detailed specifications of automatic transmissions and engine performance equipped with commercial vehicles are not released. Hence it is hard to predict where is the gear shifting point and, therefore, what is the tractive force at a certain speed. Practically, we can estimate the acceleration capability of commercial vehicles as a function of velocity and road grade from experimental data on flat terrain as Figure 1. From the driving tests of a vehicle in a nominal loading condition, we can get the relation between speed and acceleration from (2), i.e.

\[ a = \frac{1}{M}(F_t - \lambda \cdot A \cdot v^2 - \mu \cdot M \cdot g) \]  

Assuming the gear is properly shifted by the automatic transmission system, the tractive force can be approximated as a function of speed only, then the acceleration of a vehicle in a nominal loading condition can be expressed as follows:

\[ a(v)_{nominal} = \frac{(F_t(v) - \lambda \cdot A \cdot v^2)}{M_{nominal}} - \mu \cdot g \]  

Then for a different loading condition \( M \) and non-zero road grade, the acceleration capability can be described by :

\[ a(\theta, v, M)_{max} = \frac{M_{nominal}}{M} \left[ a(v)_{nominal} + \mu \cdot g \right] \]  

which could give a conservative limit for acceleration in the sense that (5) does not mean the actual physical limit of the acceleration.

By setting (5) to be zero, the maximum velocity of a vehicle can be obtained as a function of road grade and vehicle mass.

Figure 1: Typical Engine Performance Curve

Figure 2: Estimation of Acceleration Limit
3.2 Coordination Layer

A type of hybrid control system relevant to coordination layer was proposed as follows in [11]:

\[
\min_{\Omega, \mathcal{X}_{\text{ref}}} J[\mathcal{X}_{\text{ref}}(\mathcal{Q}, \mathcal{X}, t)] \quad (6)
\]

subject to

\[ C(\mathcal{Q}) \leq 0 \quad : \text{constraints} \]

\[ \mathcal{X} = f(\mathcal{X}, t, \mathbf{u}) \quad : \text{vehicle dynamics} \]

where \( \mathcal{Q} \) is a parameter set which should be determined by coordination layer, \( \mathbf{u} = u(\mathcal{X}, \mathcal{X}_{\text{ref}}, t, \mathbf{k}) \) is a feed-back control which is designed with some robust methods in the stage of regulation layer control design, and \( \mathcal{X}_{\text{ref}}(t) = g(\mathcal{X}, t, \mathcal{Q}) \) is the reference trajectory state vector which is to be determined by coordination layer.

4. Problem Formulation

4.1 Problem Statement: Mathematical Programming

Suppose that the road grade \( \Theta(x) \) is known for a given section of AHS and expressed by a function of position variable \( x \). The basic objectives of the trajectory planning in coordination layer are to minimize the maneuver time and use of control input while maximizing the safety and satisfying constraints. From the regulation layer model, constraint on velocity is obtained as a function of road grade, and lower and upper limits of acceleration are also represented as the functions of velocity and road grade. In addition to constraints on velocity and acceleration limits, maneuver time, relative velocity or distance between vehicles and other restrictions on quantities related to maneuvers can be imposed as constraints on optimization process. Using the coordination layer model in (6), the maneuver design problem for a single vehicle can thus be stated as follows:

Find a reference trajectory \( \mathcal{X}_{\text{ref}} = (x_{\text{ref}}, v_{\text{ref}}, a_{\text{ref}}) \in \mathbb{R}^3 \) and an optimal parameter set \( \mathcal{Q} \) for \( t \in [T_s, T_f] \) such that

\[
\min_{\mathcal{Q}, \mathcal{X}_{\text{ref}}} J = \min_{\mathcal{Q}, \mathcal{X}_{\text{ref}}} \int_{T_s}^{T_f} (a_1 + a_2 | a_{\text{ref}}) dt \quad (7)
\]

subject to

\[ c.1 \quad x_{\text{ref}} = v_{\text{ref}}, v_{\text{ref}} = a_{\text{ref}} \]

and

\[ c.2 \quad v_{\text{min}}(\Theta(x)) \leq v_{\text{ref}}(x) \leq v_{\text{max}}(\Theta(x)) \]

and

\[ c.3 \quad a_{\text{min}}(v, \Theta) \leq a_{\text{ref}}(v, \Theta) \leq a_{\text{max}}(v, \Theta) \]

\[ \text{or} \]

\[ c.4 \quad d_{\text{min}} \leq d_{\text{ref}} = d_{\text{init}} + \int(v_p(t) - v_{\text{ref}}(t)) dt \leq d_{\text{max}} \]

or

\[ d_{\text{ref}} = d_{\text{init}} + \int f(v_p(t) - v_{\text{ref}}(t)) dt = d_{\text{design}} \]

\[ \text{or} \]

\[ c.5 \quad v_{\text{ref}}(t) = v_{\text{ref}}(t) - v_p(t) \leq v(t)_{\text{allow}} \quad \text{for} \quad t \in [T_s, T_f] \]

\[ \text{or} \]

\[ c.6 \quad v(T_f) = v_{\text{design}} \quad \text{or} \quad a(T_f) = a_{\text{design}} \]

where \( T_s \) is the maneuver start time and \( T_f \) is the maneuver end time. \( d_{\text{ref}} \) is the relative distance between vehicles. \( v_{\text{allow}} \) is allowable relative speed \([4,6]\). \( v_p \) and \( a_p \) represent the speed and acceleration of a platoon or preceding vehicle. Upper and lower limits on vehicle speed are given by:

- lower limit: \( v_{\text{min}} = \max(v_{\text{min}}(\Theta), v_{\text{min}}) \)
- upper limit: \( v_{\text{max}} = \min(v_{\text{max}}(\Theta), v_{\text{max}}) \)

Upper limits on acceleration are also given:

\[ a_{\text{max}} = \min(a_{\text{max}}(\Theta, v), a_{\text{max-limit}}) \]

\[ a_{\text{min}} = a_{\text{min-limit}} \]

where \( v_{\text{min}}, v_{\text{max}}, a_{\text{min-limit}}, a_{\text{max-limit}} \) can be set appropriately for safety reasons or vehicle maker’s suggestion.

The performance index (cost function) is chosen to be simple and piecewise linear function. The cost function uses a weighted one-norm which is convex. The first part of the cost function represents the time optimal consideration and second one is selected for minimum energy consumption. Coefficients \( \alpha_i \)'s are weighting factors. The parameter set \( \mathcal{Q} \) contains \( T \) and \( T_f \) however the elements of \( \mathcal{Q} \) are not necessarily restricted to these two parameters but can be extended to other relevant parameters according to target maneuver. If we choose the linear constrains then the problem yields a linear program which can be solved in finite number of computation \([5]\). However if the constraints are chosen to be non-linear, then the problem becomes the nonlinear program, which is more difficult to solve.

4.2 Discretization for Numerical Solution

As stated in the previous section, the minimization of a cost function with constraints for maneuver design yields a mathematical programming problem.

To get the numerical solution, the system needs to be discretized. Let the state and input vectors at i-th step are of the following form:

\[
\mathbf{x}_i^{\text{ref}} = [x_i^{\text{ref}}, v_i^{\text{ref}}, a_i^{\text{ref}}]^T \quad (8)
\]

Suppose the time range is discretized into N steps. The complete mathematical program can be formulated as follows:

Find a reference trajectory \( \mathcal{X}_{\text{ref}} = (x_i^{\text{ref}}, v_i^{\text{ref}}, a_i^{\text{ref}}) \in \mathbb{R}^3 \) and an optimal parameter set \( \mathcal{Q} \) for \( i \in [1, N] \) such that

\[
\min_{\mathcal{Q}, \mathcal{X}_{\text{ref}}} \sum_{i=1}^{N-1} \left( \alpha_1 + \alpha_2 \cdot | a_{\text{ref}} | \right) \Delta t \quad (9)
\]

subject to

\[ d.1 \quad x_i^{\text{ref}} = x_{i-1}^{\text{ref}} + \Delta t \cdot v_{i-1}^{\text{ref}}, v_i^{\text{ref}} = v_{i-1}^{\text{ref}} + \Delta t \cdot a_{i-1}^{\text{ref}} \]

and
4.3 Multiple Vehicles

The ultimate goal of the coordination layer is to coordinate the maneuvers of multiple vehicles keeping feasibility and optimality. For the maneuvers in which multiple vehicles are involved such as merging into highway [12], the mathematical program in (9) should be modified to include cost functions and constraints for multiple vehicles. Now for K vehicles, (9) can be extended:

\[
\begin{align*}
\text{dc.2} & \quad \nu_{\min} (\theta(x^j)) \leq \nu_{\text{ref}}^j \leq \nu_{\max} (\theta(x^j)) \\
\text{dc.3} & \quad a_{\min} (\nu_{\text{ref}}^j, \theta(x^j)) \leq a_{\text{ref}}^j \leq a_{\max} (\nu_{\text{ref}}^j, \theta(x^j)) \\
\text{dc.4} & \quad d_{\min} \leq d_{\text{ref}}^k = d_{\text{init}} + \sum_{i=1}^{k-1} (\nu_{i}^j - \nu_{\text{ref}}^j) \cdot \Delta t \leq d_{\max} \\
\text{dc.5} & \quad \nu_{\text{relative}}^j = \nu_{\text{ref}}^j - \nu_{p}^j \leq \nu_{\allow}^j \\
\text{dc.6} & \quad v_{\text{ref}} = v_{\text{design}}^N \quad \text{or} \quad a_{\text{ref}}^N = a_{\text{design}}^N \\
\text{dc.7} & \quad N \cdot \Delta t \leq t_{\max} \quad \text{or} \quad N \cdot \Delta t = t_{\text{design}} \\
\text{dc.8} & \quad \left| a^j - a^{j-1} \right| \leq \frac{\text{jerk}_{\max}}{\Delta t} \\
\text{dc.9} & \quad \text{other necessary constraints}
\end{align*}
\]

where time interval \( \Delta t \) is determined by \( \Delta t = \frac{T_f - T_s}{N} \).

5. Maneuver Design Strategy

5.1 Overview

The nonlinear program in section 4 can be solved by proper numerical method. Many useful numerical methods have been developed [8]. General procedure for solving maneuver design problem is as follows:

Step 1. Assuming the road grade to be zero everywhere, select an initial estimate of parameter set \( \Omega \) and reference trajectory which are solutions of (7)

Step 2. Impose the road grade constraint and calculate the solution iteratively until optimality conditions and constraints are satisfied. The process may require several iterations.

5.2 Parameterization of Trajectory for Initial Estimate

The maneuver design algorithm should produce a result promptly to guarantee the real-time implementation of AHS. The convergence of the numerical solution of nonlinear programming problem is highly dependent on proper selection of the initial value(s) and the simplicity of cost and constraint functions

Longitudinal maneuvers basically can be described as the combination of acceleration and deceleration. To perform a join maneuver, for example, we need to accelerate first then decelerate and for split maneuver, deceleration is followed by acceleration. Bang-bang type acceleration profile with maximum allowable amplitude guarantees the minimum time maneuver, however this should be avoided in practice, particularly for heavy duty vehicles.

Considering this, we use bang-bang type control just for the initial guess since any fixed time maneuver trajectory is not feasible if the minimum time counterpart violate the constraint on maneuver time. For the mathematical simplicity, we can parameterize bang-bang type acceleration as Figure 3. To avoid infinite jerk and keep the jerk within the allowable limit, parameter \( p_1 \) is introduced. This is basically an interpolation around the discontinuous points. Then the reference trajectory can be represented by a set of parameters and therefore the optimization procedure is taken over a parameter set only. This yields a convex, piece-wise linear cost function which can be transformed into a linear form by introducing slack variables and additional constraints. Following algorithm can be used to find an initial value of the original problem in section 4:

**Algorithm for Selection of Initial Estimate**
Step 1. Assuming that the road grade to be zero, i.e. flat terrain, and the maximum values of acceleration and deceleration are given by constant values.

Step 2. Using simple parameterization for acceleration and deceleration profile, construct a nonlinear programming problem.

Step 3. Solve the problem to get the parameters.

6. Software

The optimization problem, which was formulated in the previous section, can be easily solved by MINOS solver. MINOS solver is a software package for solving large-scale optimization problems. MINOS can process large numbers of nonlinear constraints. The nonlinear functions should be smooth but need not be convex. For problems with nonlinear constraints, MINOS uses a sparse SLC algorithm (a projected Lagrangian method, related to Robinson's method). It solves a sequence of sub problems in which the constraints are linearized and the objective is an augmented Lagrangian (involving all nonlinear functions). Convergence is rapid near a solution.

MINOS solver requires the standard MPS-format as input [1]. To generate the MPS-format file efficiently, we have used the AMPL-language [8]. Implementing the constraints in AMPL is straightforward and requires minimum translation from the mathematical formulation given in section 4. The forms of the problem and the constraints are defined in a model file, while the parameter values are in a separate data file. AMPL combines the model and data to an MPS-file, which is then solved by MINOS solver.

We then specify the system matrices and other optional parameters to generate AMPL input file. An AMPL calls the solver and write the resulting solution to an output file.

7. Case Study : Join Maneuver

Due to the limitation of software, a simple example is considered. For multiple vehicle coordination, research is under going and will be presented next paper. As an example for the maneuver design strategy, consider the join maneuver between two vehicles. The goal of the control law in a join maneuver is to decrease the initial relative distance between the preceding platoon (or vehicle) and the following platoon (or vehicle) to a desired inter-platoon spacing \( d_{\text{design}} \). The relative speed should be zero at the end of the join maneuver. In this example, two vehicles are cruising with a prescribed velocity \( V_p = 60 \text{mph} \) initially and the following vehicle tries to decrease inter-vehicle distance by 3m while the leading vehicle moves with constant velocity \( V_p \). Relation between acceleration and velocity in (4) was obtained from a driving test.

Step 1. Initial Estimate Using Parameterized Trajectory

For the initial value estimate, we use the similar parameterization as Figure 3. Then total maneuver time will be \( (p_1 + 4 \cdot p_3 + 2 \cdot p_4 + p_5) \) and the integration of the absolute value of acceleration be \( 2 \cdot p_2 \cdot (p_3 + p_4) \). The relevant cost function and constraints in (6) can thus be expressed using the parameters \( p_1 \sim p_5 \):

- Cost function:
  \[ \alpha_1 \cdot (p_1 + 4 \cdot p_3 + 2 \cdot p_4 + p_5) + \alpha_2 \cdot p_2 \cdot (p_3 + p_4) \]

- Constraints
  c.1 \( d_{\text{init}} + p_2 \cdot (p_3 + p_4) \cdot (2 \cdot p_3 + p_4 + 2 \cdot p_5) = d_{\text{design}} \)
  c.2 \( p_2 \leq a_{\text{max}} \)
  c.3 \( V_{\text{min}} \leq V_0 - p_2 \cdot (p_4 + p_5) \text{ and } V_0 + p_2 \cdot (p_4 + p_5) \leq V_{\text{max}} \)
  c.4 jerk < jerk_{\text{max}}

Figure 4 shows an numerical example of the initial estimate of reference acceleration profile and Figure 5 is the corresponding velocity profile for join maneuver assuming zero road grade. \( V_{\text{max}} \) is set to be 65mph. Initially, the nominal maximum acceleration is assumed to be 0.3 m/s\(^2\) and jerk_{\text{max}} is 0.3 m/s\(^3\).

Figure 4: Acceleration Profile : Initial Estimate

Figure 5: Velocity Profile : Initial Estimate

Step 2. Iterative Computation with Constraints

Then suppose that there is road grade as shown in Figure 6. Figure 7 represent initial velocity profile and velocity limit and \( v_{\text{max}}(\theta) \) is obtained using (4)–(5). Initial estimate of velocity does not violate the constraints. However, in Figure 8, initial estimate of acceleration shows the possible actuator saturation in the position range 50-100(m). Figure 9 – Figure 10 are new
design values obtained from numerical iteration. Both velocity and acceleration satisfy the constraints on them.

Figure 6: Road Grade

Figure 7: Initial Velocity Estimate Vs Velocity Limit

Figure 8: Initial Acceleration Estimate VS Acceleration Limit

Figure 9: Designed Velocity VS Velocity Limit

Figure 10: Designed Acceleration VS Acceleration Limit

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