Trip Assignment

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Background

- Simulation based models
  - based on some predefined rules
  - simulate the existing traffic pattern and the effects of traffic control policies of interest
  - But cannot tell what is “good” and how can be better
  - High manpower cost

- Analytical models
  - built entirely on mathematical equations and inequalities
  - Simple but useful for transport planning due to their relative simplicity and lower manpower costs for implementation

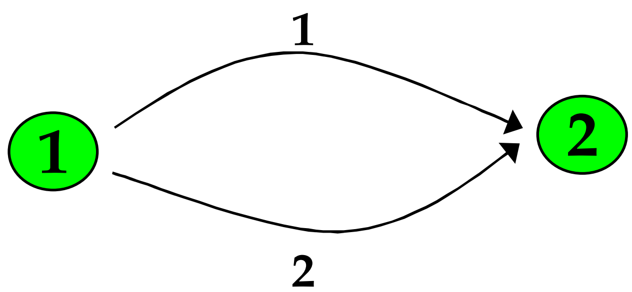
Outline

- Static model
- Dynamic models
  - Day-to-day dynamics
  - Within day dynamics
- Conclusions
- Discussion - possible future work

Traffic assignment

- Supply-demand analysis
  - Supply – characteristics and performance of transport system
    - E.g. traffic flow, travel time …
  - Demand – travelers' behavior
    - E.g. travelers' usage, mode choice, route choice, …
- Each traveler chooses to reduce individual travel cost
- *Equilibrium (steady state)* reached when costs experienced by everyone are equal
  - No one can further reduce his / her cost

Equilibrium



Travel demand:

$$J_{od} = 12 (veh / hr)$$

Travel cost:

$$t_1 = 10 + 2x_1 (\text{min})$$
$$t_2 = 5 + x_2 (\text{min})$$

Equilibrium solution

$$x_1^* = 2.333\,(\text{veh/min})$$
$$x_2^* = 9.667\,(\text{veh/min})$$
$$t_1^* = 14.667\,(\text{min})$$
$$t_2^* = 14.667\,(\text{min})$$

Total system cost:

$$Z = \sum_{\forall a} x_a t_a$$

Total system travel time (cost) = $Z = 176$ ← **Minimum?**

- Equilibrium does not lead to the best use of the system
- System optimum
  - traffic is assigned such that the overall benefit of the whole system is maximized
    - e.g. minimize the total system cost
  - provides a bound on how the road system can be optimally used

System Optimum– Formulation

- Minimize the total system cost

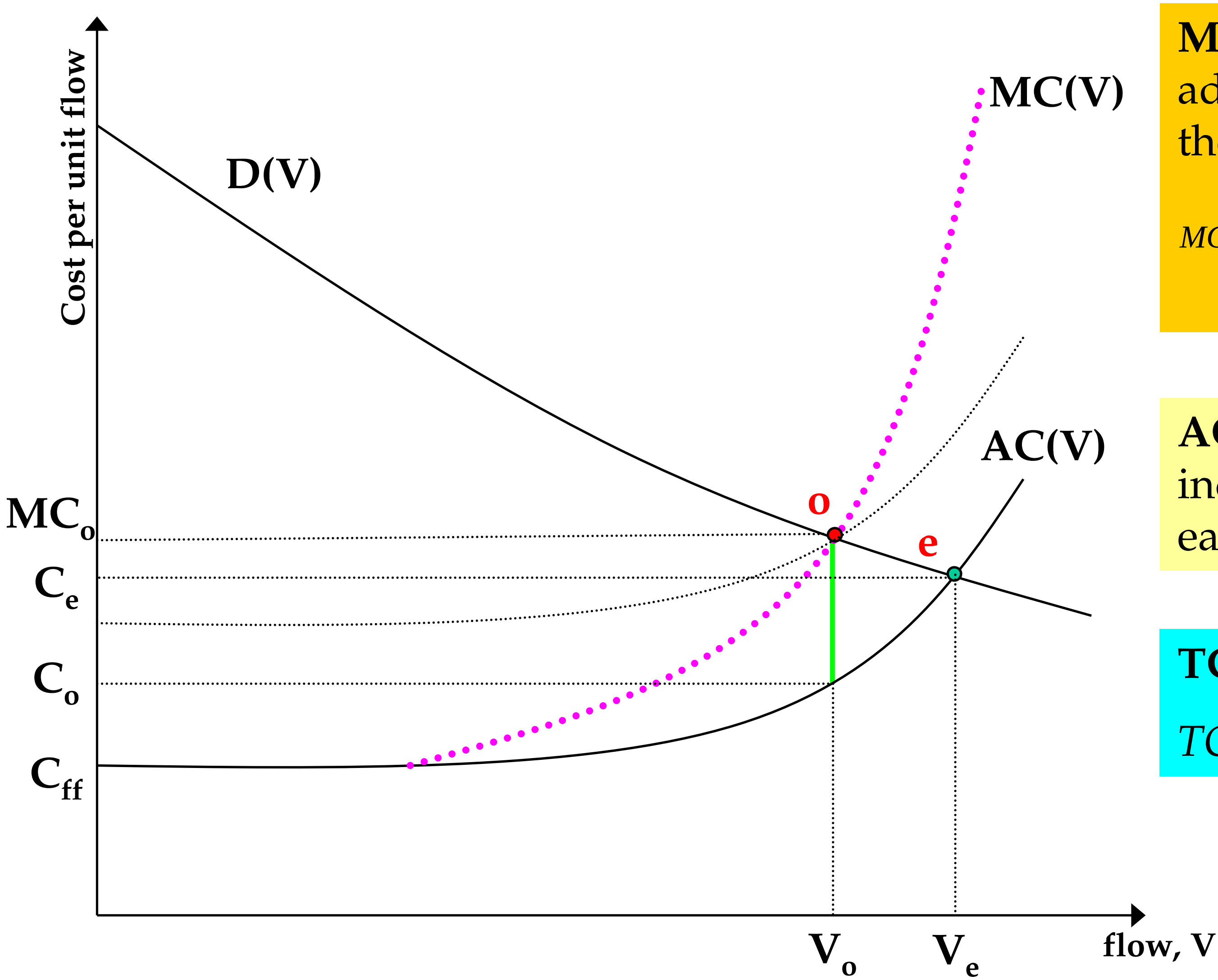
$$\min_{x_a} Z = \sum_{\forall a} x_a t_a$$

  - Subject to constraints of
    - Flow conservation
    - Total traffic volume
    - Non-negativity of traffic

- *Optimality* reached when *marginal costs* experienced by everyone are equal

$$\frac{\partial Z}{\partial x_a} = t_a + x_a \frac{\partial t_a}{\partial x_a}$$

Marginal cost pricing



MC: Additional cost of adding one extra traveller to the network

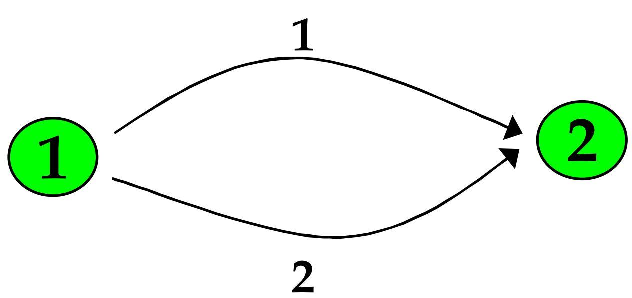
$$MC(V) = \frac{dTC(V)}{dV} = AC(V) + V\frac{dAC(V)}{dV}$$

AC: Average travel cost incurred by each traveller at each level of flow

TC: total system cost

$TC(V) = [AC(V)]V$

System optimum



Travel demand:

$J_{od} = 12$ veh/min

Travel cost:

$t_1 = 10 + 2x_1$ (min)

$t_2 = 5 + x_2$ (min)

Marginal Travel cost:

$$\tilde{t}_1 = t_1 + x_1 \frac{dt_1}{dx_1} = 10 + 4x_1 \text{ (min)}$$

$$\tilde{t}_2 = t_2 + x_2 \frac{dt_2}{dx_2} = 5 + 2x_2 \text{ (min)}$$

Equilibrium solution

$x_1^* = 2.333$ (veh/min)

$x_2^* = 9.667$ (veh/min)

$t_1^* = 14.667$ (min)

$t_2^* = 14.667$ (min)

System optimal solution

$x_1^* = 3.167$ (veh/min)

$x_2^* = 8.833$ (veh/min)

$t_1^* = 16.333$ (min)

$t_2^* = 13.833$ (min)

$\tilde{t}_1^* = 22.667$ (min)

$\tilde{t}_2^* = 22.667$ (min)

Total system travel time (cost) = $Z' = 173.914$

Limitations 1

- Disregard how an equilibrium (or system optimal) state is actually reached over time

Day-to-day dynamics – Equilibrium

- "Day-to-day" travelers' behavior adjustments and the evolution of the overall system towards equilibrium
  - E.g. Horowitz (1984), Cantarella and Cascetta (1995), and Watling (1999)
- Fischer et al. (2006) considered how travelers compute and learn an equilibrium more efficiently based on simple sampling and adaptation policies
  - -> ATIS deployment

Day-to-day dynamics – System optimum

- Friesz et al. (2004) considers the travel demand is *elastic* to travel cost
  - Tolling strategy over "days" to maximize the "travelers' surplus" in a planning horizon
- Yang et al (2007) showed that with the conventional *static* marginal cost, the overall system performance may not be montomically improved over "days" during the transition before reaching its ultimate system optimal state
  - "Dynamic marginal toll"
  - "steepest-descent toll"

Limitations 2

- On the side of characteristics of traffic,
  - neglect the temporal variation of congestion

- On the side of travel demand
  - Do not consider the departure time choices of travellers
    - an important choice dimension to consider
  - Peak shifting – common and *real* in urban area
    - Shown empirically by the case study of San Francisco – Oakland Bay bridge (Small, 1982)

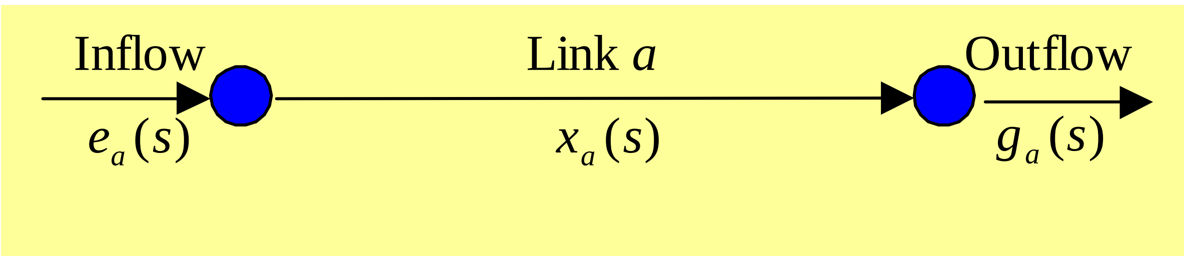
Small, K. (1982) The scheduling of consumer activities: work trips. *American Economics Review*, **72**(3), 467-479.

Within-day dynamic trip assignment

- Supply: temporal variation of traffic congestion and travel cost
- Demand: travelers' responses
  - route choices
  - departure time choices
  - route and departure time choices

Supply dynamics

- A general model of traffic



$$\frac{dx_a(s)}{ds} = e_a(s) - g_a(s)$$

Link travel time = linear function of volume of link traffic

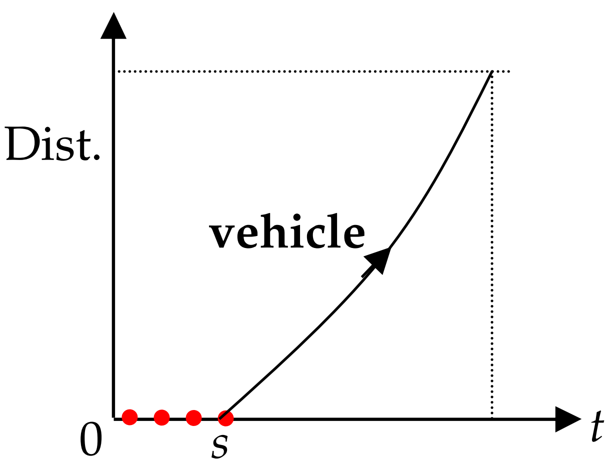
=free flow travel time + (amount of traffic) / capacity

Traffic Models – Desirable properties

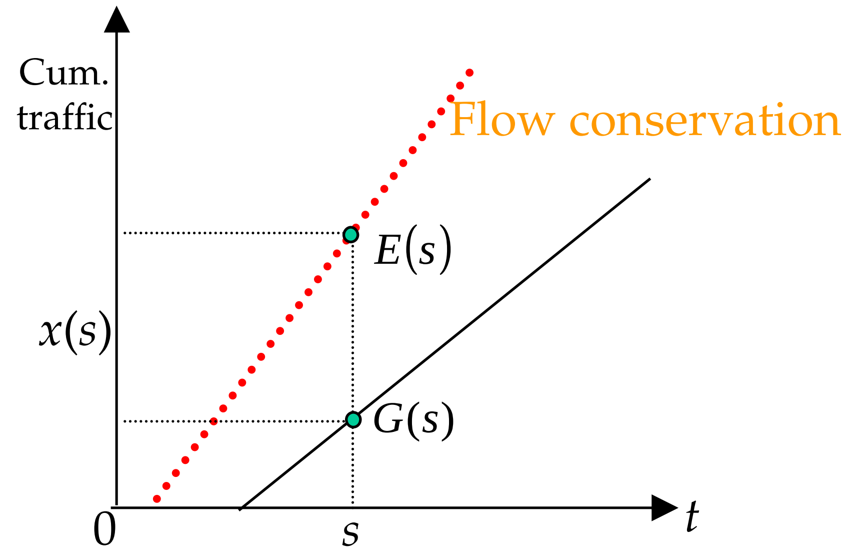
- Non-negativity $e \geq 0 \Rightarrow x \geq 0, g \geq 0$
- Flow conservation (link and node)

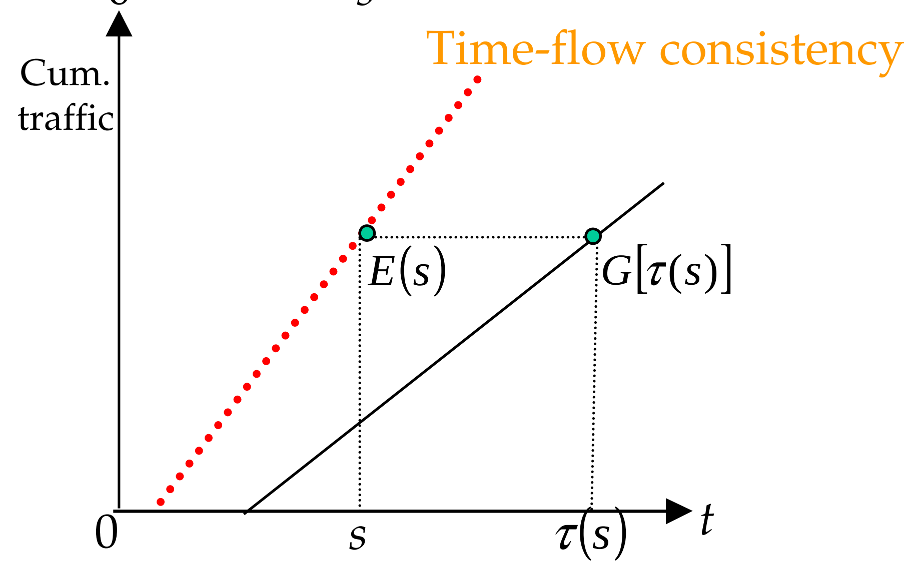
  $x(s) = E(s) - G(s)$ or $\dot{x}(s) = e(s) - g(s)$
- First-in-first-out $\dot{\tau}(s) \geq 0$
- Time-flow consistency

  $E(s) = G[\tau(s)]$ or $e(s) = g[\tau(s)]\,\dot{\tau}(s)$
- Causality



$\tau(s)$



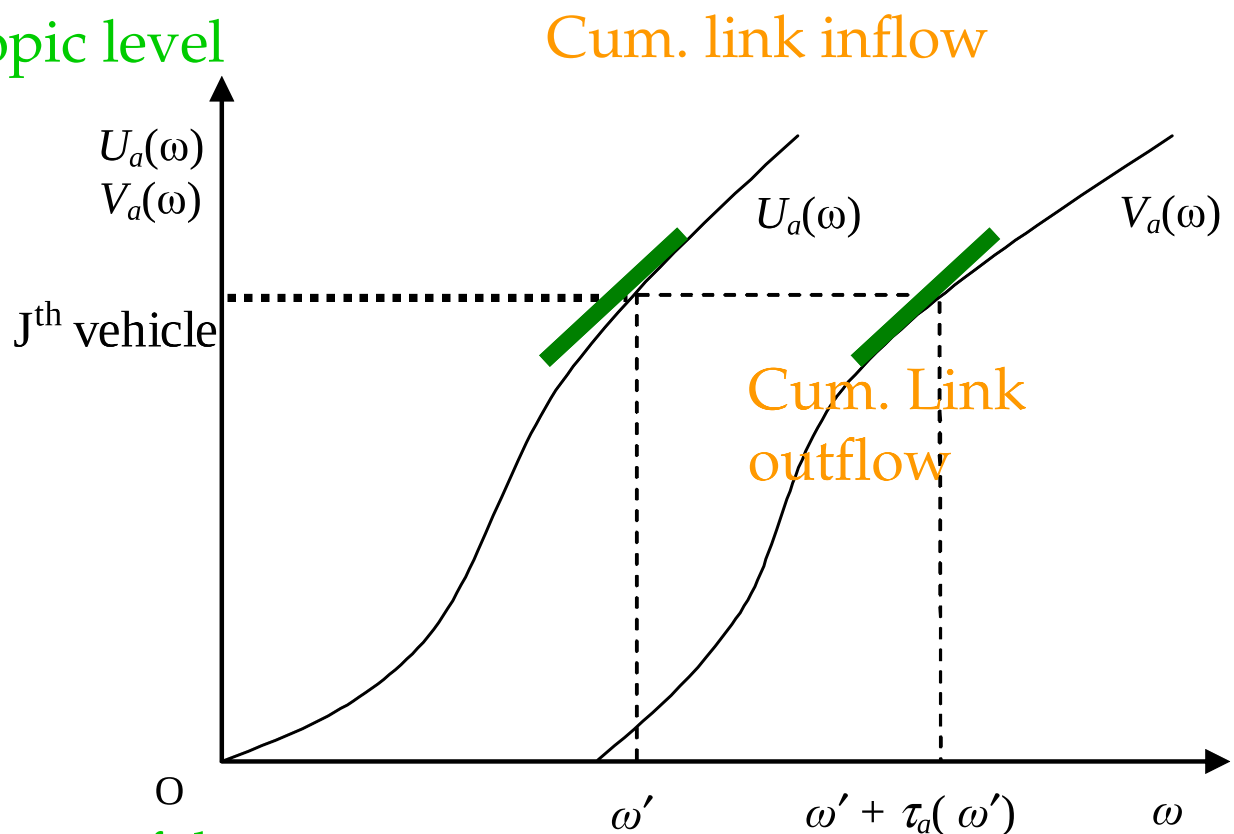


Time-flow consistency

- FIFO and flow conservation helps defining travel time and time flow consistency at the macroscopic level

$$U_a(\omega) = V_a\left(\omega + \tau_a(\omega)\right)$$

$$v_a\left(\omega + \tau_a(\omega)\right) = \frac{u_a(\omega)}{1 + \frac{d\tau_a(\omega)}{d\omega}}$$



Travel time is related to the tangent of the cum. curve,

i.e. Flow

Demand dynamics

- Each traveler chooses to minimize individual travel cost
  - Total travel cost = travel time + *time-specific costs*
  - Time-specific costs - the preferences of:
    - arrival times
    - departure times

Departure time specific cost

- The value of time to travellers staying at the origin

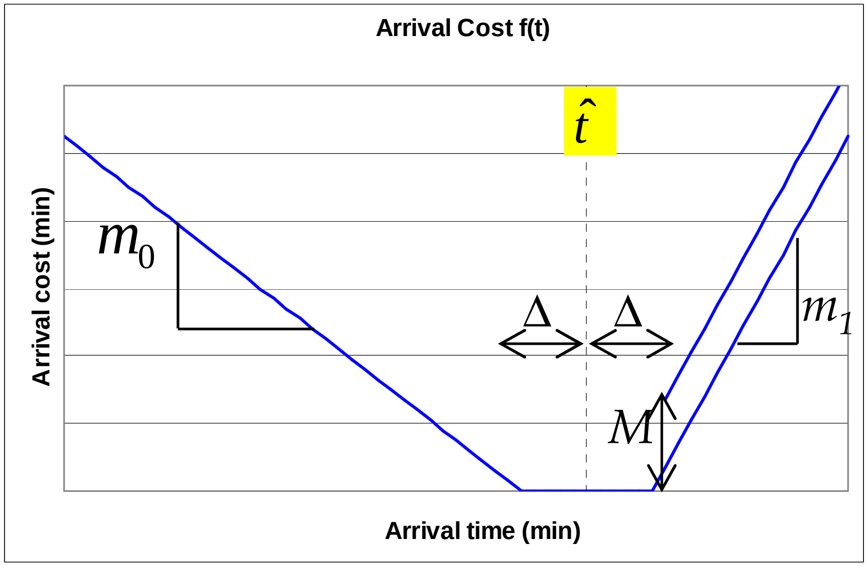
- Non-increasing function
  - travellers would gain continuing benefits (i.e. lower cost) from remaining at their origin.
  - they are drawn to their destination by a need to attend and hence to travel.

Arrival time specific cost

- A cost associated with traveller's arrival time at the destination of a journey

- The general form of this cost function is

$$f(t)=\begin{cases} m_0\left(\hat{t}-\Delta-t\right) & \left(t\le\hat{t}-\Delta\right) \\ 0 & \left(\hat{t}-\Delta\le t\le\hat{t}+\Delta\right) \\ m_1\left(t-\hat{t}-\Delta\right)+M & \left(t\ge\hat{t}+\Delta\right) \end{cases}$$



- Small (1982) reported the empirical finding that $m_1 \approx 2m_0$

Demand dynamics

- Each traveler chooses to minimize individual travel cost
  - Total travel cost = travel time + *time-specific costs*
  - Time-specific costs - the preferences of:
    - arrival times
    - departure times
- *Wardrop Equilibrium* reached when costs experienced by everyone are equal
  - No one can further reduce his / her cost

Formulations of DUE

- Mathematical programming (MP)
  - e.g Janson, 1991; Han and Heydecker, 2006
- Non-linear complementarity problem (NCP)
  - e.g. Wie et al., 2002
- Fixed point problem (FPP)
  - e.g. Addison and Heydecker, 1993; Heydecker and Addison, 1996
- *Variational inequality (VIP)*
  - e.g. Friesz et al., 1993; Ran and Boyce, 1996; Szeto and Lo, 2002, 2004; Nie and Zhang, 2007
  - Link-based formulation: Ban et el. (2007) for *point-queue* model

Variational Inequality (VI) Formulation

- To find $\mathbf{f}^*$ such that:

Route travel time vector

Solution set

Route flow vector

$$(\mathbf{f} - \mathbf{f}^*)^{\mathrm{T}} \cdot \mathbf{n}^* \geq 0, \forall \mathbf{f} \in \Omega$$

$$\Omega = \left\{ \mathbf{f} \mid \mathbf{Af} = \mathbf{q}, \mathbf{f} \in R_+^{n_2} \right\}$$

demand vector

- A Solution exists if
  - The feasible set of inflow is closed and convex
  - $\mathbf{n}$ is a continuous mapping of $\mathbf{f}$
- Uniqueness further requires $\mathbf{n}$ is a strictly monotone mapping

Cell-based DUE

- Lo and Szeto formulated a CTM based dynamic user equilibrium assignment by using VIP
  - with route choice (Lo and Szeto, 2002)
  - with combined route and departure time choice (Szeto and Lo, 2004)
- The formulation was solved by using a projection method developed by Han and Lo (2003)
  - convergent if the solution set of the VI problem is nonempty
- Weaknesses: solving cell-based TA is computationally expensive and may not be suitable for large scale computations
  - Friesz and Bernstein (2000) also pointed out that CTM is difficult to analyse mathematically because the outflow function in CTM is piecewise and is not differentiable with respect to its state variable

Properties of DUE

Properties	Point queue	Physical queue
Flow conservation	satisfy	
Monotonicity	Monotonic	Non-monotonic
Uniqueness	Multiple solution (w.r.t. route flow)	
FIFO	may not satisfy	satisfy
Time-flow consistency	may not satisfy	satisfy
Causality	may not satisfy	satisfy
Continuity of travel time w.r.t. flow	Continuous	Possibly discontinuous
Existence of UE solution	Always exist a solution	May not exist if travel time is discontinuous

Dynamic system optimal assignment

- Equilibrium does not lead to the best use of the system
- System optimum
  - traffic is assigned such that the overall benefit of the whole system is maximized
    - e.g. minimize the total system cost
  - provides a bound on how the road system can be optimally used
  - Insights on designing and implementing control policies

Dynamic system optimal assignment – Challenges

- A dynamic optimisation problem
- Solution: an optimal profile over time (not a single value!)
- Have to capture properties of traffic dynamics:
  - Flow conservation,
  - Flow – travel time consistency,
  - causality

  in a plausible way

Dynamic system optimization Control theoretic approach

- E.g. Merchant and Nemhauser, 1978 a,b; Papageorgiou, 1990; Wie et al., 1994; Wie et al., 1995 a, b; Yang and Huang, 1997; Friesz et al. 2007
- Minimize the total system cost

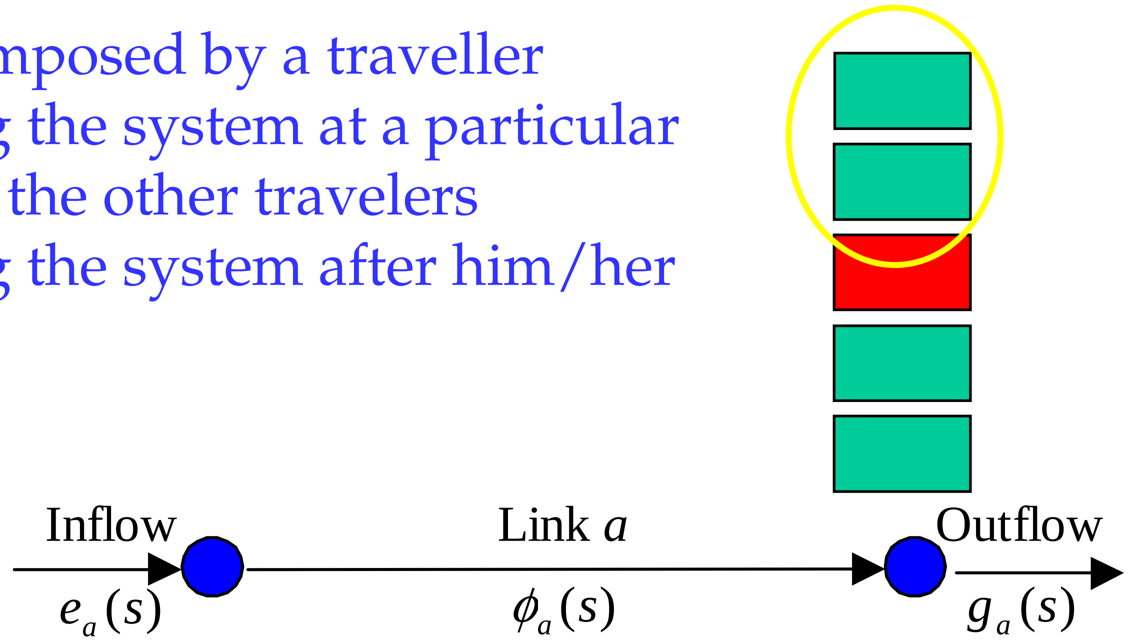
$$\min_{e_a^*(s)} Z = \sum_{\forall a} \int_0^T C_a(s) e_a(s) ds$$

  - Subject to constraints of
    - Flow conservation
    - *Consistency of travel time and traffic flow*
    - Non-negativity of traffic
    - Satisfy a predefined total travel demand

- *Optimality* reached when travellers' individual cost plus *dynamic externality* associated with everyone are equal

Dynamic externality – within day

- Cost imposed by a traveller entering the system at a particular time on the other travelers entering the system after him/her



Solving DSO control problem - Gradient based approach

- Step 0: Set the initial values for the control, state, and costate
- Step 1: Calculate the state variable and *externality* forward in time
- Step 2: Calculate the gradient / step size for control
- Step 3: Update the control
- Step 4: Calculate the costate variable, based on the updated control, backward in time
- Step 5: Convergence test

Cell-based DSO – Linear Programming approach

- Ziliaskopoulos (2000) formulated a CTM based dynamic system optimal assignment with route choice for one-to-many networks by using the linear programming (LP) approach:-
  - minimize the total system travel time (i.e. occupancies in each cell at each time interval)
  - Control variables: route inflow
  - State variables: cell occupancy
  - Dual variables: marginal cost with cell occupancy
- Necessary and sufficient conditions were derived and proven.
- Ziliaskopoulos (2000) assumed that traffic could be held *everywhere* at *every time* for the benefit of the whole system
  - Such *holding back* problem was criticized to be unrealistic but should be able to remedy
- Li et al. (2003); and Kalafatas and Peeta (2006) reformulated Ziliaskopoulos (2000) by using a *graph theoretic* approach. It seems simplifying the computation

Conclusions

- Supply-demand Framework for modeling and managing (time-varying) network traffic

Discussion - Possible future work

- Data collection / calibration
  - California PeMS dataset
- Control strategies
  - Dynamic tolls (e.g. marginal pricing), ramp metering, signal control …
- Medium/large real networks
  - Physical features and interaction of traffic (e.g. spilling back)
- Characteristics of travelers
  - Imperfect information (-> provision of ATIS)
  - Heterogeneity (in terms of travelers'/vehicles' characteristics.. etc), which implies *equity* related issues