The SWARM algorithms

SWARM 1

- Coordinated strategy
- Has 2 primary functions: forecast and apportionment.
- Freeway is divided into sections, each section limited downstream by a bottleneck.
- Regulates density at the bottleneck section, keeping it below the *estimated saturation density.*
- Saturation density is estimated daily using a 2nd order polynomial fit on the fundamental diagram.
- Ramp metering rates are set based on forecasted bottleneck densities. Forecast is made with a Kalman filter (?)

Forecast model: Assume the current time is t. Then the basic forecast model is that density at time t+1 equals the density at time t plus b, where b represents the forecasted slope of the density trend at time t. The key to the forecast is to obtain the best possible estimate of b. To accomplish this objective, two steps are required; first estimate the slope of the line in the interval (t-h) to t using a simple linear curve technique: second apply a Kalman filter (see Appendix B for test results) using the observed values of density obtained during the same time interval. The Kalman filter produces a new value of b that is then used to forecast the density at the next time period. This procedure is reiterated until T_{crit} is reached. The result is a forecasted value of density.

It is felt that, considering the frequency of update, the SWARM 1 algorithm will be able to anticipate the onset of congestion very effectively.

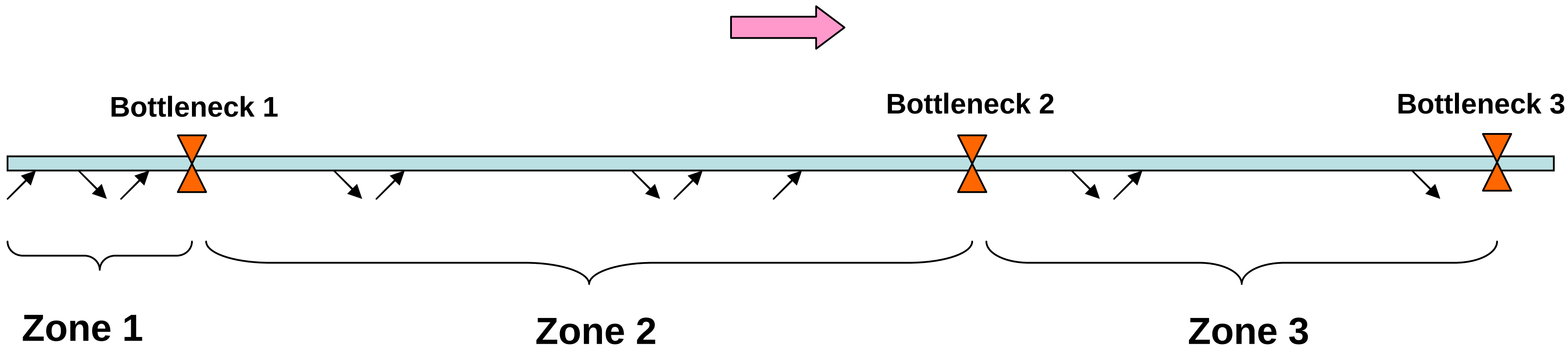
SWARM 2

- Two local strategies (2a and 2b)
- Only one of the two can be used, perhaps in combination with SWARM 1.

SWARM 2a is theoretically based on headway theory and uses a density function to compute local metering rates. It attempts to maintain headway by optimizing density such that maximum flow can be accomplished. As such the algorithm requires no saturation densities for operation.

SWARM 2b is an algorithm based upon computing the number of vehicles stored between two VDSs. This algorithm requires measurements of volume and speed at the VDSs, tables containing distances between VDSs, and storage saturation values. These requirements are shown in the following figure depicting a Storage Zone.

[Freeway segmentation] SWARM 1



- The freeway is divided at the bottlenecks into zones.
- Each zone refers to its downstream bottleneck.

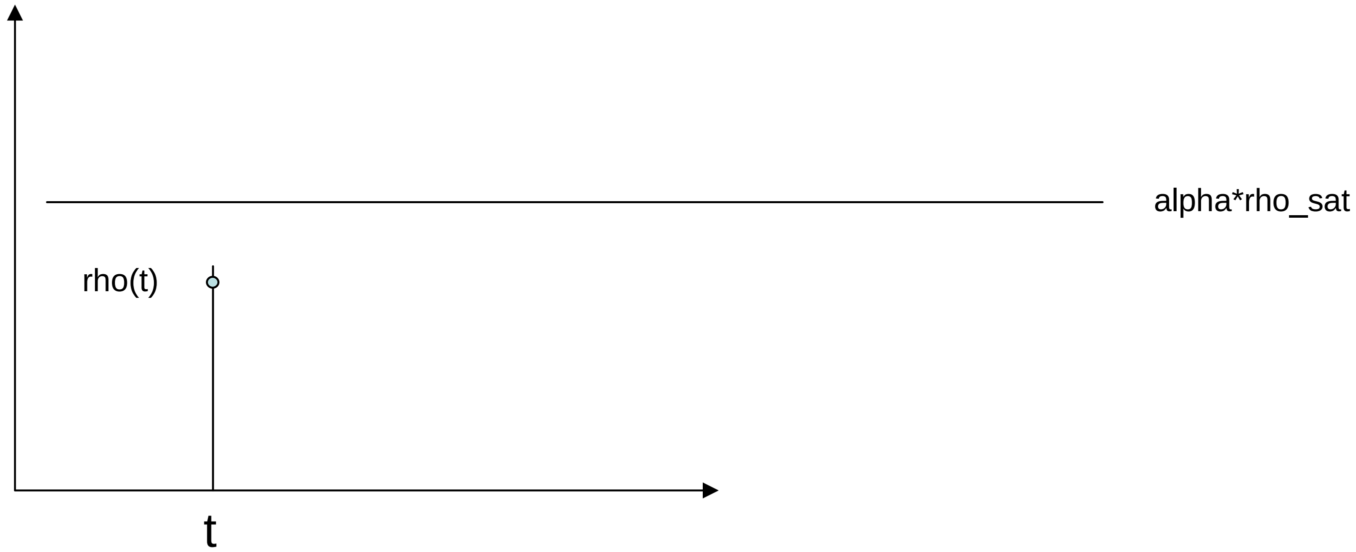
[Kalman Filter] SWARM 1

- A Kalman filter is used to estimate the future bottleneck density
- The a-priori estimate is used as the “observation”!

NET code.

```
for ( i=0; i<FORECAST_LEAD_TIME; i++ ){

     SW_BOTTLENECK[k].Tcrit = SW_BOTTLENECK[k].Tcrit + 1;

     temp = SW_BOTTLENECK[k].pre_mean_density_forecast;
     SW_BOTTLENECK[k].pre_mean_density_forecast = SW_BOTTLENECK[k].post_mean_density_forecast * MODEL_PARA_A
                                              + SW_BOTTLENECK[k].norm_station_density_slope_forecast * MODEL_PARA_B;

    SW_BOTTLENECK[k].pre_variance_density_forecast =
                  pow( MODEL_PARA_A, (double)2 ) * SW_BOTTLENECK[k].post_variance_density_forecast
         + pow( MODEL_PARA_G, (double)2 ) * MODEL_PARA_Q;

    observation_station_density = SW_BOTTLENECK[k].pre_mean_density_forecast;

    SW_BOTTLENECK[k].post_variance_density_forecast = 1/( 1/ SW_BOTTLENECK[k].pre_variance_density_forecast
                                                  + MODEL_PARA_H * MODEL_PARA_H/MODEL_PARA_R );

    SW_BOTTLENECK[k].post_mean_density_forecast = SW_BOTTLENECK[k].pre_mean_density_forecast
               + SW_BOTTLENECK[k].post_variance_density_forecast *( MODEL_PARA_H/MODEL_PARA_R )
               * ( observation_station_density - MODEL_PARA_H *SW_BOTTLENECK[k].pre_mean_density_forecast );

    for ( j=0; j < SLOPE_SAMPLE_SIZE - 1; j++ )
       SW_BOTTLENECK[k].slope_density_forecast[j] = SW_BOTTLENECK[k].slope_density_forecast[j+1];
       SW_BOTTLENECK[k].slope_density_forecast[SLOPE_SAMPLE_SIZE-1] = observation_station_density;

    SW_BOTTLENECK[k].norm_station_density_slope_forecast = density_slope_calculator_forecast(k);

}
```

[Required current density] SWARM 1

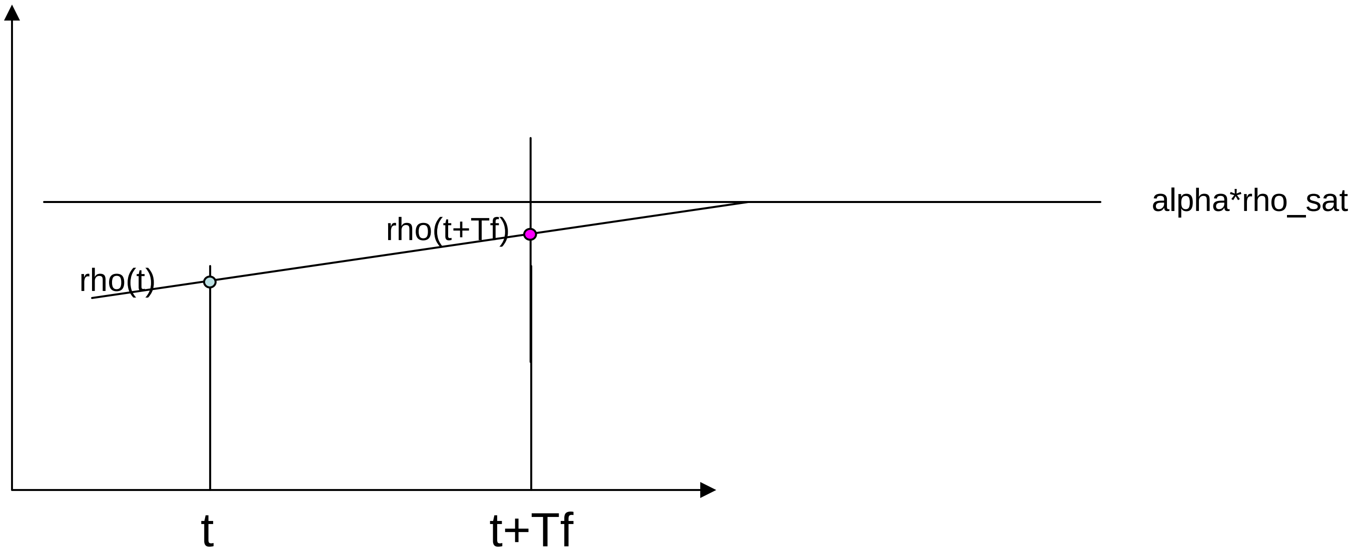


For each zone, at time t we check whether the measured density at the bottleneck is above or below alpha*rho_sat, where rho_sat is the critical density and alpha<1.

We want to calculate rho_req(t), the current density required so that the bottleneck will not become saturated Tf seconds in the future.

If rho(t) > alpha*rho_sat → rho_req(t) = alpha*rho_sat

The current value of density must be less than saturation density to proceed with a forecast. If the current density is greater than or equal to saturation density, the forecasting portion of the algorithm is skipped.

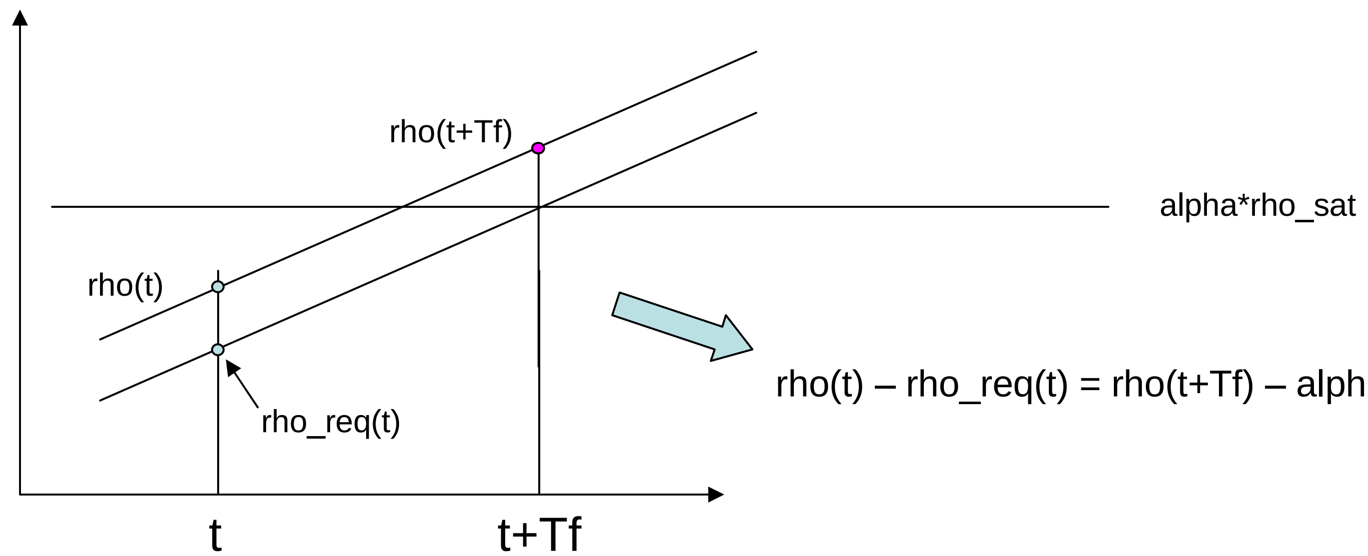


If rho(t) > alpha*rho_sat → rho_req(t) = alpha*rho_sat

Otherwise estimate rho(t+Tf)
If rho(t+Tf) < alpha*rho_sat → rho_req(t) = alpha*rho_sat

If the forecasted density is less than saturation density or the current density is greater than or equal to saturation density then the required bottleneck density equals saturation density.

[Required current density] SWARM 1



rho(t) – rho_req(t) = rho(t+Tf) – alpha*rho_sat

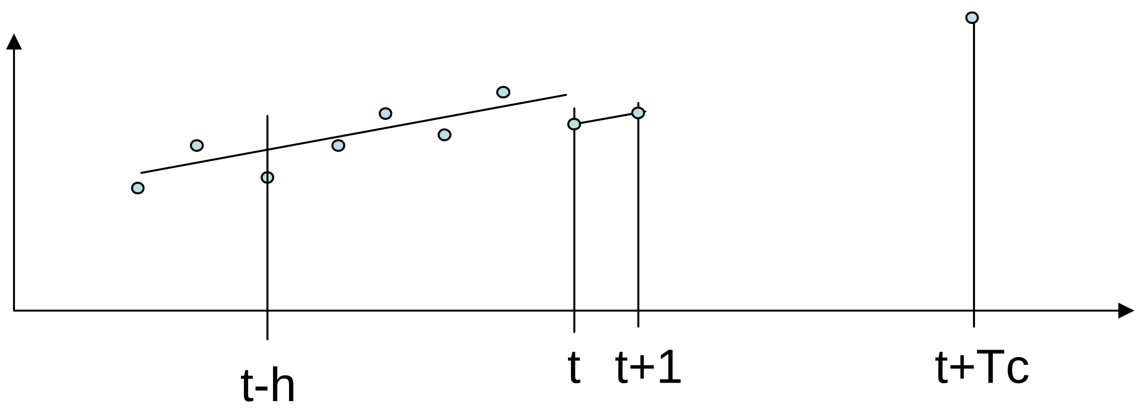
If rho(t) > alpha*rho_sat → rho_req(t) = alpha*rho_sat

Otherwise estimate rho(t+Tf)
If rho(t+Tf) < alpha*rho_sat → rho_req(t) = alpha*rho_sat

Otherwise
rho_req(t) = rho(t) – rho(t+Tf) - alpha*rho_sat

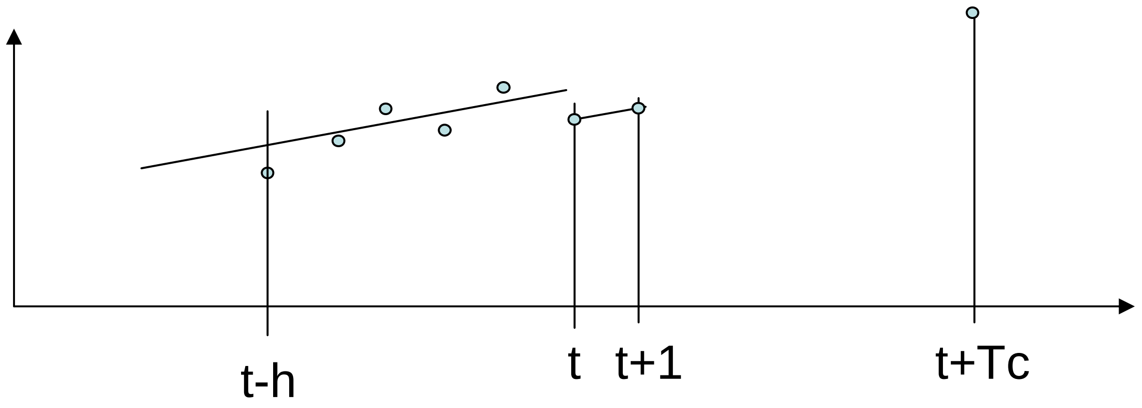
units?

If the forecasted value of density exceeds the bottleneck's saturation density then the <u>required bottleneck density</u> will be computed as follows: current density minus $1/T_{crit}$ times the amount of excess density (above saturation density) computed above.



Forecast model: Assume the current time is t. Then the basic forecast model is that density at time t+1 equals the density at time t plus b, where b represents the forecasted slope of the density trend at time t. The key to the forecast is to obtain the best possible estimate of b. To accomplish this objective, two steps are required; first estimate the slope of the line in the interval (t-h) to t using a simple linear curve technique: second apply a Kalman filter (see Appendix B for test results) using the observed values of density obtained during the same time interval. The Kalman filter produces a new value of b that is then used to forecast the density at the next time period. This procedure is reiterated until T_{crit} is reached. The result is a forecasted value of density.

It is felt that, considering the frequency of update, the SWARM 1 algorithm will be able to anticipate the onset of congestion very effectively.



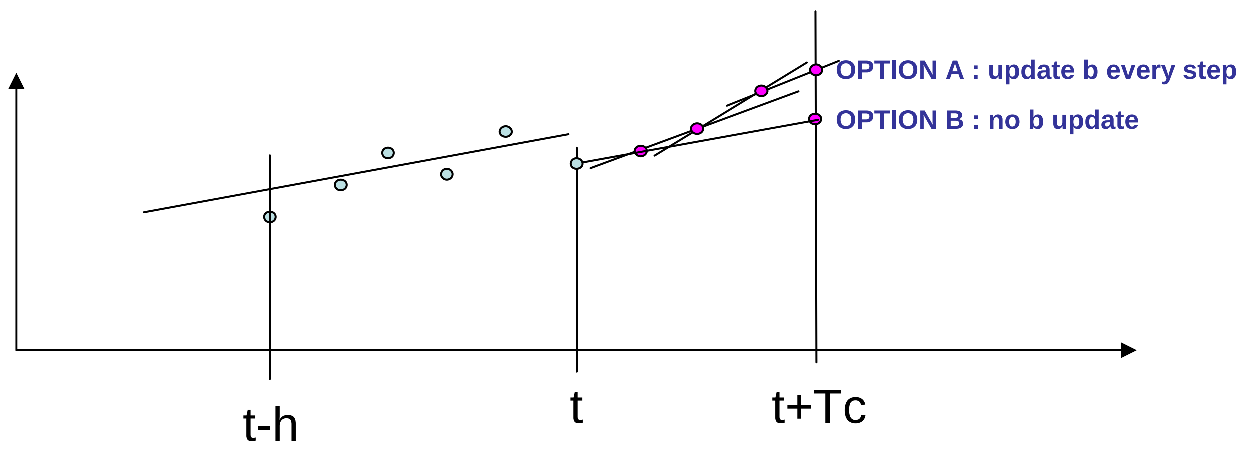
- b = least squares on data in [t-h…t] : $b = \dfrac{h\sum k x_k - \sum k \sum x_k}{h\sum k^2 - \left(\sum k\right)^2}$
- Model: $x_{k+1} = x_k + b + w_k$
  $y_k = x_k + v_k$
- "Forecast" densities in [t-h…t] using Kalman filter:

$$\begin{cases} \hat{x}_k^- = \hat{x}_{k-1} + b \\ \hat{P}_k^- = \hat{P}_{k-1} + Q \\ K_k = \hat{P}_k^- / (\hat{P}_k^- + R) \\ \hat{P}_k = (1 - K_k)\hat{P}_k^- \\ \hat{x}_k = \hat{x}_k^- + K_k(z_k - \hat{x}_k^-) \end{cases}$$

- Forecast densities in [t-h…t] using "Kalman filter":

$$\begin{cases} \hat{x}_k^- = \hat{x}_{k-1} + b \\ \hat{P}_k^- = \hat{P}_{k-1} + Q \\ K_k = \hat{P}_k^- / (\hat{P}_k^- + R) \\ \hat{P}_k = (1 - K_k)\hat{P}_k^- \\ \hat{x}_k = \hat{x}_k^- + K_k(z_k - \hat{x}_k^-) \end{cases}$$

$z_k = \hat{x}_k^-$!


OPTION A : update b every step
OPTION B : no b update
t-h
t
t+Tc

Once we have calculated rho_req(t) for each zone, we need to find the metering rates that will bring the current density at the bottleneck to rho_req.

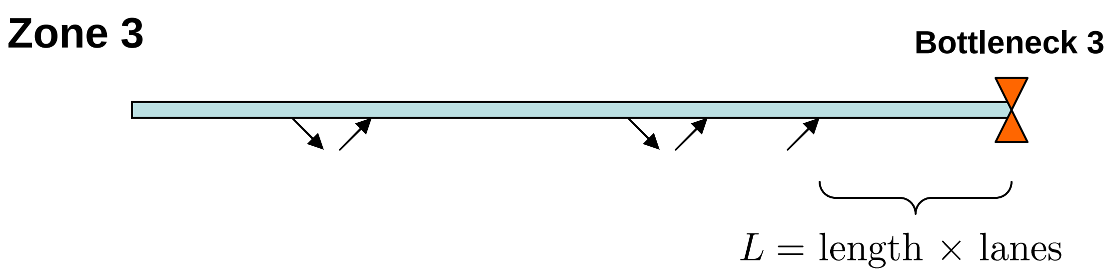
We do this by converting the density difference into an adjustment to the metering rate, and then apportioning this adjustment to the different ramps within the zone.

The algorithm involves:

rex … excess onramp metering rate

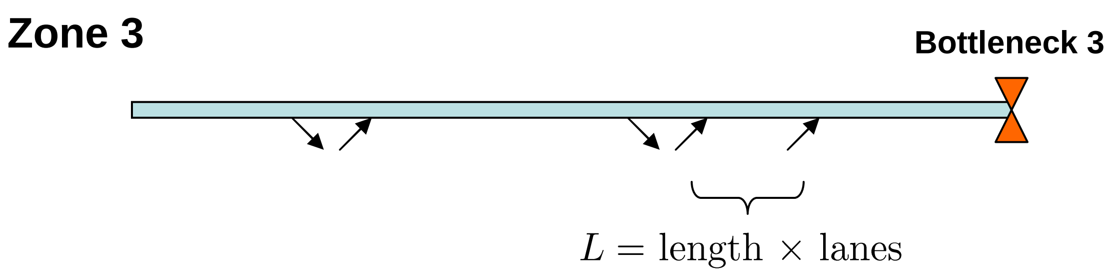
rdes …. Desired onramp metering rate

rc … actual onramp metering rate



$$r_{ex} = 0$$
$$\Delta r = L(\rho_b(t) - \rho_{req})$$
$$r_{des} = r(t) - \Delta r - r_{ex}$$
$$r_c = \min\left(\max\left(r_{des}, r_{min}\right), r_{max}\right)$$
$$r_{ex} = \phi(r_c - r_{des})$$

- ϕ is an intra-zone propagation factor.
- r_{ex} can be positive or negative.

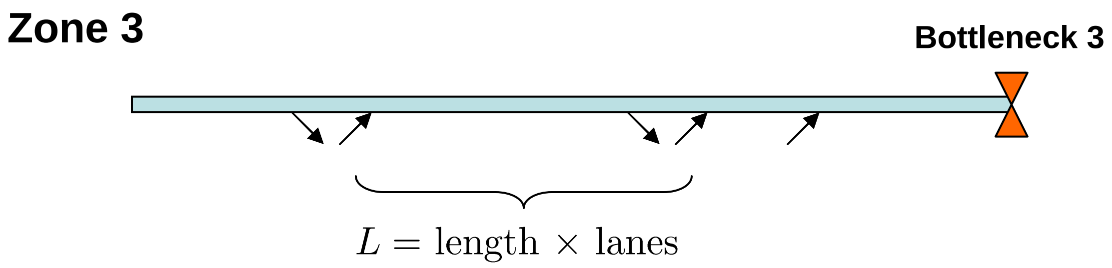


$$\Delta r = L(\rho_b(t) - \rho_{req})$$

$$r_{des} = r(t) - \Delta r - r_{ex}$$

$$r_c = \min\left(\max\left(r_{des}, r_{min}\right), r_{max}\right)$$

$$r_{ex} = \phi(r_c - r_{des})$$



$$\Delta r = L(\rho_b(t) - \rho_{req})$$
$$r_{des} = r(t) - \Delta r - r_{ex}$$
$$r_c = \min\left(\max\left(r_{des}, r_{min}\right), r_{max}\right)$$
$$r_{ex} = \psi\phi(r_c - r_{des})$$

- ψ is an inter-zone propagation factor.

SWARM 2a

Given

- j = VDS upstream of ramp station number (numbered upstream)
- t = Sample time interval
- n_j = Number of Lanes at VDS_j
- $D_{j,t}$ = Density at VDS_j at time t (Veh/Mile/Lane)
- $S_{j,t}$ = Station Speed at VDS_j at time t (Miles/Hour)
- λ_j = Vehicle length at VDS_j (feet)
- ρ_j = Postmile at VDS_j (miles)
- ξ_j = Target speed reduction (Feet/Sec); initial value = 1
- $Rmax_j$ = Maximum metering rate at ramp j (Vehicles per minute)
- $Rmin_j$ = Minimum metering rate at ramp j (Vehicles per minute)

Then the following is computed for each ramp at time t

Speed in $\frac{ft}{sec}$ $\quad \varsigma_{j,t} = S_{j,t} * \frac{5280}{3600}$ (Feet/Sec)

Headway at VDS_j $\quad H_{j,t} = \frac{5280 - D_{j,t} * \lambda_j}{D_{j,t} * \varsigma_{j,t}}$ (seconds)

Speed Reduction per mile $\quad \Delta s = \frac{\xi_j}{\rho_j - \rho_{j-1}}$

Then Compute $\quad \tau_{j,t} = 19 - 2 * n_j * (\rho_j - \rho_{j+1}) \left[\frac{5280}{(\varsigma_{j,t} - \Delta s) * H_{j,t} + \lambda_k} - D_{j,t} \right]$

Metering Rate $\quad R_j, t = Minimum(Maximum(\tau_{j,t}, Rmin_j), Rmax_j)$

SWARM 2a

SWARM 2b

<u>Given</u>

j = First Good VDS upstream of metered ramp j
$j+1$ = First Good VDS downstream of metered ramp j
Zone j = The section of freeway between VDS_j and VDS_{j+1}
n_j = Number of Lanes at VDS_j
$V_{j,t}$ = Volume at VDS_j at time t
$SumE_{j,t}$ = Sum of Volumes at all entrance ramps between VDS_j and VDS_{j+1} at time t
$SumEx_{j,t}$ = Sum of Volumes at all exit ramps between VDS_j and VDS_{j+1} at time t
w_j = Weighing factor for final Storage computation (Initial value = 0.1)
$D_{j,t}$ = Density at VDS_j at time t (Veh/Mile/Lane)
$\epsilon^{e \to d}$ = Percentage reducton in density LOS E to LOS D
$Dsat_j$ = Saturation Density at VDS_j at time t (Veh/Mile/Lane)
ρ_j = Postmile at VDS_j (miles)
$Rmax_j$ = Maximum metering rate at ramp j (Vehicles per minute)
$Rmin_j$ = Minimum metering rate at ramp j (Vehicles per minute)

Then the following is computed for each ramp at time t

Estimated Storage in Zone j $\quad \hat{\sigma}_{j,t} = \dfrac{n_j\,(\rho_j - \rho_{j+1})(D_{j,t} + D_{j+1,t})}{2}$

Net Change in Zone j $\quad \Delta_t = V_{j,t} - V_{j+1,t} + SumE_{j,t} - SumEx_{j,t}$

Cumulative Storage in Zone j $\quad \overline{\sigma_{j,t}} = \sigma_{j,t-1} + \Delta_t$ $\quad$ ($\sigma_{j,t-1}$ defined below)

IF any ramp detectors are failed in Zone j THEN

$\quad k = 1$

ELSE

$\quad k = w_j$

ENDIF

Storage in Zone j $\quad \sigma_{j,t} = k * \hat{\sigma}_{j,t} + (1-k) * \overline{\sigma_{j,t}}$

Critial Storage in Zone j $\quad \sigma^{crit}_{j,t} = \dfrac{n_j\,(\rho_j - \rho_{j+1})(Dsat_j + Dsat_{j+1})}{2}$

Max Available Capacity in Zone j $\quad C^{max}_{j,t} = \sigma^{crit}_{j,t} - \sigma_{j,t}$

Desired Available Capacity in Zone j $\quad C^{des}_{j,t} = C^{max}_{j,t} * \epsilon^{e-d}$

IF $C^{des}_{j,t} > Rmin_j$ THEN

$\quad$ Metering Rate $\quad R_{j,t} = Minimum(C^{des}_{j,t}\,,\ Rmax_j)$

ELSE

$\quad$ Metering Rate $\quad R_{j,t} = Maximum(Minimum(C^{max}_{j,t}\,,\ Rmax_j), Rmin_j)$

ENDIF