Simulating the signal intersections in cell transmission model

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Objective: To determine the traffic flow in and out from a signalized intersection with merge and diverge.

Basic notation:

i index for signalized intersections

u index for upstream links

d index for downstream links

U Set of upstream links to the intersection

D Set of downstream links from the intersection

Variables to be determined:

S_u traffic flowing from link u into the intersection

y_d traffic flowing to link d from the intersection

1. Basic formulation

For simple intersection without turnings, Lo (1999) showed that the effect of a traffic signal can be simulated in cell transmission model (CTM) by formulating the capacity Q_u of the upstream link u just before the signal light as a time varying binary variable:

$$Q_u(t) = \begin{cases} Q_{\max} & \text{if } t \in \text{green phase} \\ 0 & \text{if } t \in \text{red phase} \end{cases},$$

where $Q_{\max}$ is the maximum flow from u.

Hence,

$S_u(t) = \min\{n_u(t), Q_u(t), \frac{w}{v}[N_d - n_d(t)]$, and

$y_d(t) = S_u(t)$,

2. Signalized merge*

2.1 With all turnings protected

For intersections where all turning movements are protected, merging traffic will not be flowing into the downstream link at the same time. For each merging link *u*, we can determine its outflow to the intersection as,

$S_u(t) = \min\{n_u(t), Q_u(t), \frac{w}{v}[N_d - n_d(t)]$, for all merging links *u*.

Whether which upstream link is flowing is captured by the time-variant $Q_u(t)$, which determines the actively flowing link accordingly to the signal timing plan.

The inflow to the downstream link *d* of the signal light can simply be regarded as the sum of the outflows from its upstream:

$y_d(t) = \sum_U S_u(t)$,

2.2 With turnings permitted†

In general case where some turning movements are permitted rather than protected at an intersection.

* Daganzo (1995) and Lo (1999) both limited the number of merging and diverging branches not exceeding two. However, I think the formulation should still work given the merge/diverge ratio are defined properly. We may discuss further on this.

† We need further discussion on this part. I will discuss the derivation of the following expressions in the next meeting. The analysis in Daganzo (1995, pp 85 - 87) was limited to two merging links only. We have to see if it is possible to extend that. I am also thinking of using Gabriel's ACTM approach to do this.

Daganzo(1995) considered the case of a merge of two links. He came with three resulting possible cases:

<u>Case a</u>: The flow on both merging links is dictated by conditions upstream (i.e. no congestion and waves move forward). The condition for case (a) to happen will be

$$y_d(t) = \sum_U S_u(t) < \frac{w}{v}[N_d - n_d(t)],$$

and the associated flow on the merging links can be determined as

$$S_u(t) = \min\{n_u(t), Q_u(t)\}, \quad \text{for all merging links } u.$$

When the total inflow from the upstream is greater than the space that the downstream can accommodate, i.e. $y_d(t) = \sum_U S_u(t) > \frac{w}{v}[N_d - n_d(t)]$, we have the following two possible case arisen:

<u>Case b</u>: The flow on both merging links is dictated by conditions downstream (i.e. congestion on both links, waves move backward);

<u>Case c</u>: The flow from one link is dictated by upstream while the other by downstream.

To determine the flow on each merging link, we need to specify the "merging proportion" associated with each of the merging link. Define $\alpha_u(t)$ [‡], where $\sum_U \alpha_u(t) = 1$ for all time *t* and hence, $S_u(t) = \alpha_u(t) y_d(t)$.

For cases (b) and (c), Daganzo (1995) showed that

$$S_u(t) = mid\{n_u(t), \frac{w}{v}[N_d - n_d(t)] - S_{\sim u}(t), \alpha_u \frac{w}{v}[N_d - n_d(t)]\},$$

for all merging links *u*, where the notation ~*u* refers the another merging link apart from *u*.

[‡] At this point, the merging ratios are assumed to be given exogenously.

3. Signalized diverge

Define the split ratio from the intersection to its downstream diverging links d at time t be $\beta_d(t)$[§], where $\sum_D \beta_d(t) = 1$ for all time t.

Hence, $y_d(t) = \beta_d(t) S_u(t)$

In order to maintain a first-in-first-out (FIFO) queuing discipline, the sending flow from the upstream link i should be restricted when either one of the downstream diverging links is unable to accommodate its allocation of flow. This condition is maintained by

$$S_u(t) = \min \left\{ \begin{array}{c} n_u(t) \\ Q_u(t) \\ \min\limits_d \left\{ \min\{Q_d(t), \frac{w}{v}[N_d - n_d(t)]\} / \beta_d \right\} \end{array} \right\}.$$

References:

Daganzo, CF (1995) The cell transmission model, Part II: Network traffic. Transportation Research Part B, 29(2), 19-93.

Lo, H (1999) A novel traffic signal control formulation. Transportation Research Part A, 33, 433-448.

[§] At this point, the split ratios are assumed to be given exogenously.