Destination based node model

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This model is to determine the link flow coming into and out from a network node, which can either be a merging or diverging node, or a combination. The model is based on the assumption that at any branching node, the destination-specific splitting ratio can be determined exogenously (e.g., via some route choice / trip assignment models).

We define the following notation

I	set of incoming links to the node
J	set of outgoing links to the node
S	set of destinations in the network
$i \in I$	index for incoming links
$j \in J$	index for outgoing links
$s \in S$	index for destinations
$D_i(t)$	inflow on link *i* to the node at time *t*
$D_i^s(t)$	flow on link *i* heading to a destination $s \in S$ at time *t*, where $D_i(t) = \sum_{s \in S} D_i^s(t)$, and $D_i^s(t) = D_i(t)\alpha_i^s(t)$, $\alpha_i^s(t)$ is a destination-specific proportion parameter
$P_j(t)$	available space on link *j* to accommodate the outflow from node at time *t*
$b_j^s(t)$	ratio of the flow on the diverging link *j* heading to destination *s*, to the total inflow to the node, and $\sum_{j \in J} b_j^s(t) = 1, \forall s \in S$ for all times *t*.

Given the above setting, the node model is to determine:

$v_{ij}(t)$	flow from an incoming link *i* through the node to the outgoing link *j*
$v_{ij}^s(t)$	flow from an incoming link *i* through the node to the outgoing link *j* heading to destination *s*

Note that we do not track individual path flows; instead we need *destination-specific* information such as inflow $D_i^s(t)$ and split ratio $b_j^s(t)$

The node model

Let D_{ij} be the flow that is *ready* to leave from link *i* through the node to link j. It can be deduced that

$$
\begin{aligned}
D_{ij} &= \sum_s D_i^s b_j^s \\
&= \sum_s (D_i \alpha_i^s) b_j^s = D_i \sum_s \alpha_i^s b_j^s \\
&= D_i \lambda_i^j
\end{aligned}
$$

where $\lambda_i^j = \sum_s \alpha_i^s b_j^s$ is the proportion of traffic on link *i* going to link j.

Nie (2006, pp106-107) showed that the *actual* flow from *i* to *j*, denoted as v_{ij} as:

$$v_{ij} = \min\{\tilde{D}_i \lambda_i^j, \tilde{P}_j \frac{\tilde{D}_i \lambda_i^j}{\sum_l \tilde{D}_l \lambda_l^j}\}, \tag{1}$$

where

$$\tilde{D}_i = \min\{D_i, \max\{\frac{P_j}{\lambda_i^j}\}\Big|_{\forall j \in J}\}, \forall i \in I, \tag{2}$$

$$\tilde{P}_j = \min\{P_j, \sum_i D_i \lambda_i^j\}, \forall j \in J. \tag{3}$$

Furthermore, the destination based proportion of traffic from *i* to *j* can be computed as:

$$v_{ij}^s = v_{ij} \frac{\alpha_i^s b_j^s}{\sum_s \alpha_i^s b_j^s}. \tag{4}$$

Note that (4) cannot strictly enforce the proportion parameter α_i^s or the splitting ratio b_j^s for a given destination *s* under certain conditions (such as queues on some *i* or *j*).

Working Example

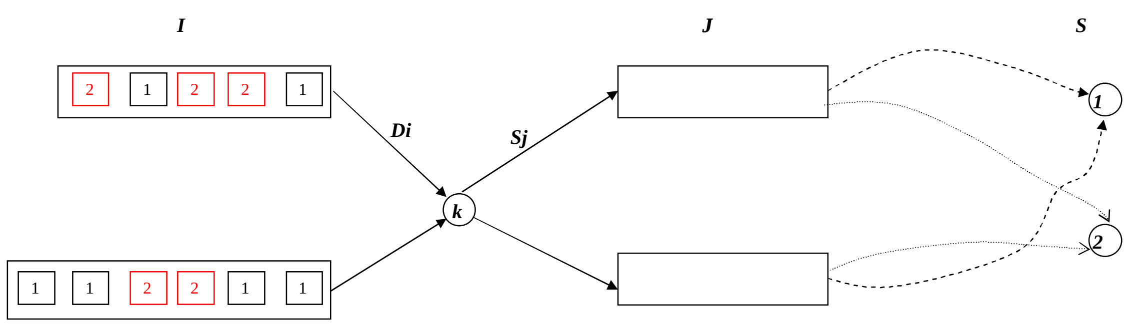

Figure 1 Destination Based Node Model

Figure 1 shows a network node in which there are two links coming in and two others going out (i.e $I = \{1,2\}$, $J = \{1,2\}$). The traffic is heading to two destinations ($S = \{1,2\}$).

Note that in the example we drop the time index "*t*" for brevity.

Given:

1. Demand: $D_1 = 5, D_2 = 6, D_1^1 = 2, D_1^2 = 3, D_2^1 = 4, D_2^2 = 2$ (i.e. $\alpha_1^1 = 0.4, \alpha_1^2 = 0.6, \alpha_2^1 = 2/3, \alpha_2^2 = 1/3$),

2. Supply: $P_1 = 4, P_2 = 8$.

3. Merging ratio: $a_1 = a_2 = 0.5$,

4. Destination-specific split ratio as $b_1^1 = 0.2, b_2^1 = 0.8, b_1^2 = 0.7, b_2^2 = 0.3$.

By solving the node model (1) – (4), we can solve v_{ij}^s which is shown in the following table:

v(i,j)		**Outgoing *j***	
		1	**2**
destination 1	**1**	0.34042553	0.4
	2	2.72340426	3.2
destination 2	**1**	1.78723404	2.1
	2	0.5106383	0.6

The *actual* α_i^s and split ratio b_j^s can also be calculated as in the following table, which are slightly different compared with the desired values.

		1	**2**
α_i^s	**1**	0.41931	0.58069
	2	0.684171	0.315829
b_j^s	**1**	0.255319	0.744681
	2	0.761905	0.238095

Reference:

Nie, Y (2006) A Variational Inequality Approach For Inferring Dynamic Origin-Destination Travel Demands. PhD thesis. University of California at Davis.