Macroscopic traffic detector model

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1 Preamble

- Loop detectors are an important component of intersection signalling systems. One of their major roles is in "actuated" systems, where an individual vehicle traversing an upstream detector (the *approach* detector) causes the signal to extend its green phase. The number of counts registered during the red phase is sometimes used to compute a *minimum green time* for the upcoming green phase.
- Discrete vehicle events are also used in coordinated and adaptive signal control algorithms.
- Aside from the approach detector (a.k.a. mid-block detector), *stopline* detectors have a function analogous to the crosswalk button: they register a call for green on a particular phase, which may shorten the length of other green phases.
- We need to devise a detector model that translates the macroscopic quantities of the model into discrete detector actuation events.
- The "hybrid" or "macro-micro" approach, in which we place a *microscopic window* over the intersection is probably more than we need. We are not interested in individual vehicle dynamics, but in the probability of observing a vehicle at a specific point in space, over a given interval of time.

2 Approach

Assumption #1: The position of the detector within the link is not considered. How we distinguish the approach detector from the stopline detector is another question, maybe they will have to be in separate links.

Assumption #2: The probability that a vehicle triggers a detector is a Poisson distribution. The probability function is:

$$f(k, \lambda) = \frac{\lambda^k e^{-\lambda}}{k!} \tag{1}$$

where k is the number of observed events (i.e. vehicle passages), λ is the expected number of events in the given time interval. The probability of registering at least one event is then:

$$1 - f(0, \lambda) = 1 - e^{-\lambda} \tag{2}$$

<u>Calculation of λ</u>.

At first glance it may seem that λ should simply be the flow. This fails however when vehicles are queued since the flow is then low, but the probability of vehicle actuation should be high. This is especially important for actuated control.

Let us consider the following formula for λ:

$$\lambda = \frac{v\Delta t + l}{l + h} \tag{3}$$

where l is the average vehicle length, Δt is the time interval, and h is the mean vehicle distance headway. This formula results from calculating the expected number of vehicle passages on a road with evenly spaced vehicles moving at a constant speed v, as shown below.

Figure 1: Δt=0

With $\Delta t = 0$, the expected number of events equals the vehicle occupancy:

$$\lambda(\Delta t = 0) = \frac{l}{l + h} \tag{4}$$

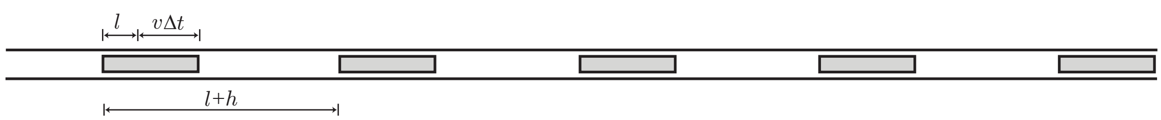

Figure 2: $v\Delta t < h$

Allowing vehicles to move, we trace their trajectory through the time interval Δt, which is assumed in the following figure to be less than h/v. The estimate of λ is now the *smeared* occupancy:

$$\lambda = \frac{v\Delta t + l}{l + h} \tag{5}$$

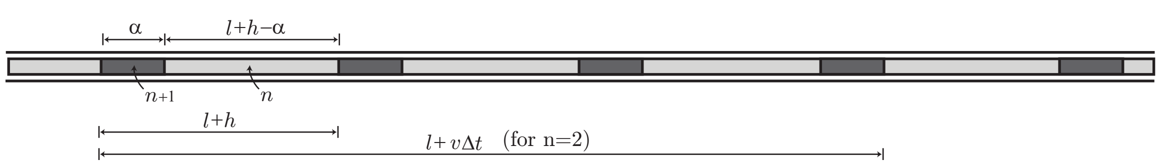

Figure 3: General case

Now, for the general case, allowing $v\Delta t$ to reach arbitrarily large values, we end up with the situation depicted in figure 3 where some reqions have been covered n times, and others $n + 1$ times.

n is the integer part of the ratio of the distanced traveled to the inter-vehicle distance.

$$n = \text{floor}\left(\frac{v\Delta t + l}{l + h}\right) \tag{6}$$

Defining α as the portion covered $n + 1$ times:

$$\alpha = l + v\Delta t - n(l + h) \tag{7}$$

we can now calculate λ:

$$\lambda = \frac{(n+1)\alpha + n(l + h - \alpha)}{l + h} \tag{8}$$

$$= \frac{n(l + h) + \alpha}{l + h} \tag{9}$$

$$= \frac{n(l + h) + l + v\Delta t - n(l + h)}{l + h} \tag{10}$$

$$= \frac{v\Delta t + l}{l + h} \tag{11}$$

Note 1. Because headway is not a macroscopic quantity, the formula actually implemented should be

$$\lambda = (v\Delta t + l)\rho \tag{12}$$

where $\rho = 1/(l + h)$ is the link density. The average vehicle length is a constant parameter.

Note 2. The assumption of zero detector length can be relaxed. For a detector with length d the formula for λ becomes:

$$\lambda = \frac{v\Delta t + l + d}{l + h} = (v\Delta t + l + d)\rho \tag{13}$$

(This should be checked.)